The Midpoint Formulas (Chapter 1)

On a Number Line:

The coordinate of the midpoint M of $\overline{AB}$ is (a + b)/2

In the Coordinate Plane:

Given AB where A(x1, y1) and B(x2, y2), the coordinates of the midpoint of AB are $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

The Distance Formula (Chapter 1)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Properties of Congruence (Chapter 2)

Reflexive Property : DE = DE $\angle D = \angle D$

Symmetric Property : If DE = FG, then FG = DE.

If $\angle D = \angle E$, then $\angle E = \angle D$.

Transitive Property : If DE = FG and FG = JK, then DE = JK

If $\angle D = \angle E$ and $\angle E = \angle F$, then $\angle D = \angle F$.

Law of Detachment (Chapter 2)

If $p \rightarrow q$ is true and p is true, the q is true.

Law of Syllogism (Chapter 2)

If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is true.

Summary of Related If-Then Statements

Given statement:	If p, then q.
Contrapositive:	If not q, then not p.
Converse:	If q, then p.
Inverse:	If not p, then not q.

A statement and its contrapositive are logically equivalent.

A statement is not logically equivalent to its converse or to its inverse.

Six Ways to prove Two Lines are Parallel (Chapter 3)

1. Show that a pair of corresponding angles are congruent.
   Postulate 3-2
2. Show that a pair of alternate interior angles are congruent.
   Theorem 3-4
3. Show that a pair of same-side interior angles are supplementary.
   Theorem 3-5
4. Show that a pair of alternate exterior angles are congruent.
   Theorem 3-6
5. Show that both lines are parallel to a third line.
   Theorem 3-7
6. In a plane show that both lines are perpendicular to a third line.
   Theorem 3-8

A way to Prove Two Segments or Two Angles Congruent (Cpt 4)

1. Identify two triangles in which the two segments or angles are corresponding parts.
2. Prove that the triangles are congruent.
3. State that the two parts are congruent, using the reason:

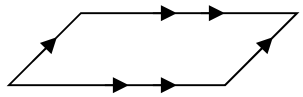
   Corr. Parts of $\cong \triangle$s are $\cong$.
   (Corresponding parts of congruent triangles are congruent.)
   Usually written CPCTC or CPCT.

Summary of Ways to Prove Two Triangles Congruent (Chapter 4)

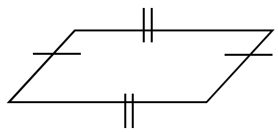
All triangles: SSS SAS ASA AAS
Right triangles: HL

Five ways to prove that a Quadrilateral is a Parallelogram (Chapter 6)

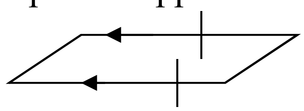
1. Show that both pairs of opposite sides are parallel.



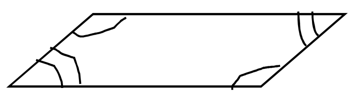
2. Show that both pairs of opposite sides are congruent.



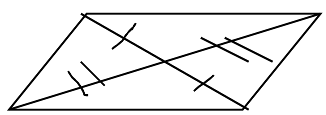
3. Show that one pair of opposite sides are both congruent and parallel.



4. Show that both pairs of opposite angles are congruent.



5. Show that the diagonals bisect each other.



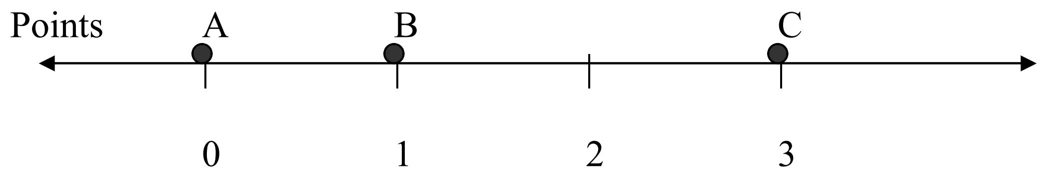
Properties of Inequality (Chapter 5??)

If $a > b$ and $c \geq d$, then $a + c > b + d$.
If $a > b$ and $c > 0$, then $ac > bc$ and $a/c > b/c$.
If $a > b$ and $c < 0$, then $ac < bc$ and $a/c < b/c$.
If $a > b$ and $b > c$, then $a > c$.
Comparison Property of Inequality.
If $a = b + c$ and $c > 0$, then $a > b$.

Theorems and Postulates

Postulate 1-5 **Ruler Postulate**

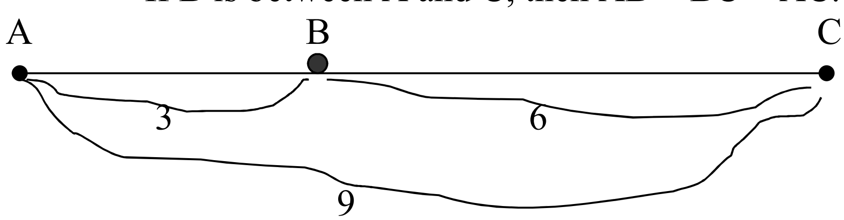
1. The points on a line can be paired with the real numbers in such a way that any two points can have coordinates 0 and 1.
2. Once a coordinate system has been chosen this way, the distance between any two points equals the absolute value of the difference of their coordinates. (page 12)



The distance from point B to point C is | 1 – 3| = |-2| = 2

Postulate 1-6 **Segment Addition Postulate**

If B is between A and C, then AB + BC = AC.



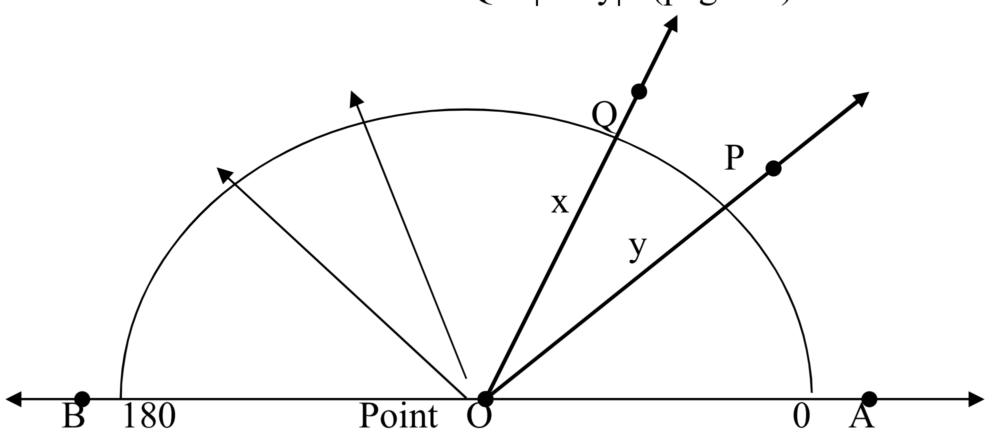
3 + 6 = 9

Postulate 1-7 **Protractor Postulate**

On $\overleftrightarrow{AB}$ in a given plane, choose any point O between A and B. Consider $\overrightarrow{OA}$ and $\overrightarrow{OB}$ and all the rays that can be drawn from O on one side of $\overleftrightarrow{AB}$. These rays can be paired with the real numbers from 0 to 180 in such a way that:

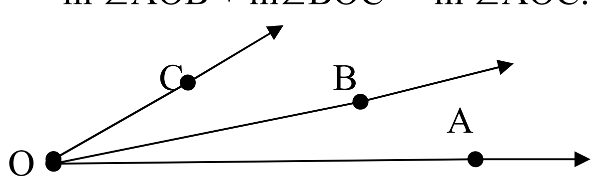
a. $\overrightarrow{OA}$ is paired with 0, and $\overrightarrow{OB}$ with 180.

b. If $\overrightarrow{OP}$ is paired with x, and $\overrightarrow{OQ}$ with y, then m ∠POQ = |x – y|. (page 18)



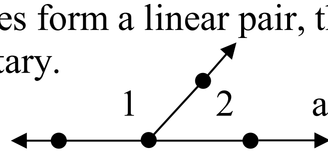
Postulate 1-8 **Angle Addition Postulate**

If point B is in the interior of ∠AOC, then
m ∠AOB + m∠BOC = m ∠AOC.



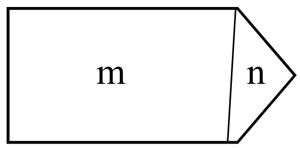
Postulate 1-9 **Linear Pair Postulate**

If two angles form a linear pair, then they are supplementary.



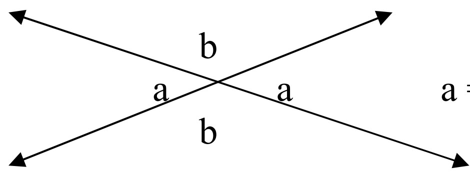
angle 1 + angle 2 are supplementary

Postulate 1-10 **Area Addition Postulate**
The area of a region is the sum of the areas of its nonoverlapping parts.

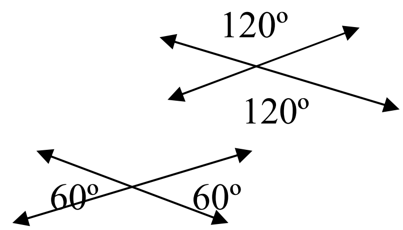


Area m + Area n = total area.

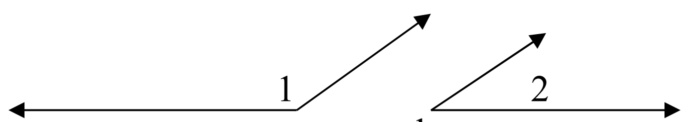
Theorem 2-1 **Vertical Angles Theorem**
Vertical angles are congruent.



a = a and b = b

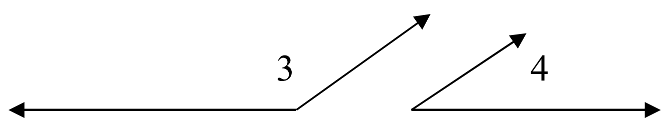


Theorem 2-2 **Congruent Supplements Theorem**
If two angles are supplements of congruent angles (or of the same angle), then the two angles are congruent.



supplements

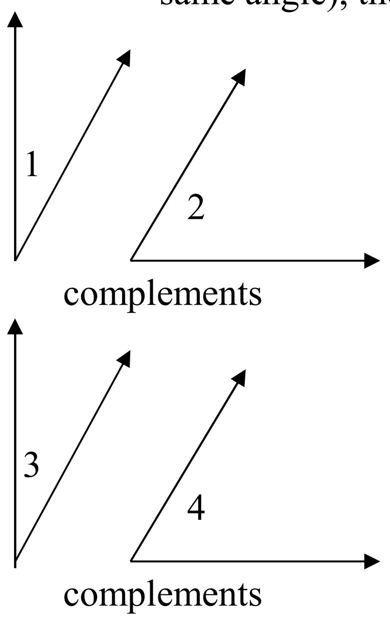
If 1 = 3



supplements

then 2 = 4

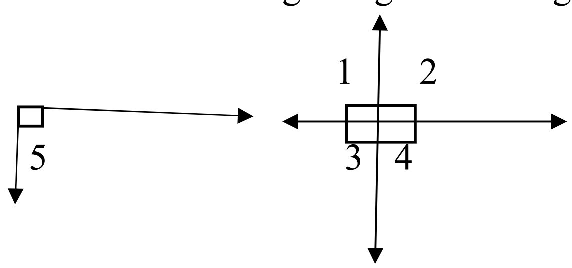
Theorem 2-3 **Congruent Complements Theorem**
If two angles are complements of congruent angles (or of the same angle), then the two angles are congruent.



If 1 = 3

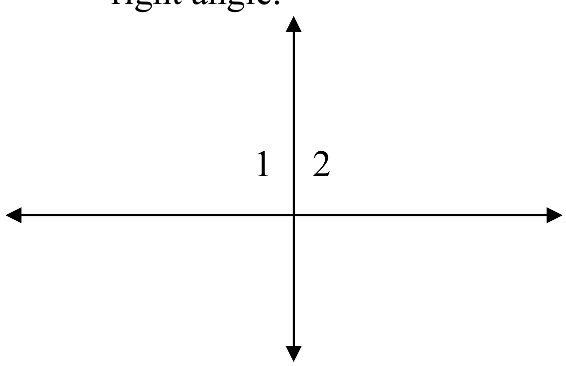
Then 2 = 4.

Theorem **2-4** All right angles are congruent.



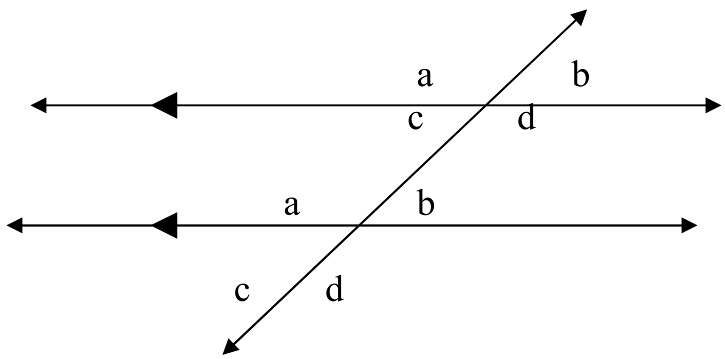
∠1 = ∠2 = ∠3 = ∠4 = ∠5

Theorem **2-5** If two angles are congruent and supplementary, then each is a right angle.

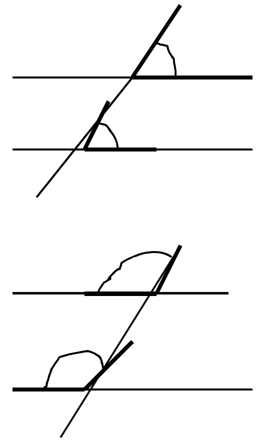


If ∠1 = ∠2, and
1 and 2 are supplementary,
then ∠ 1 and ∠ 2 are right angles.

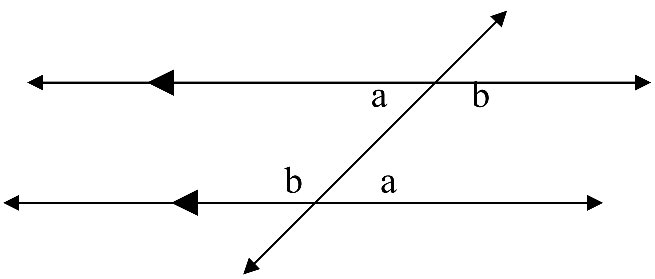
Postulate 3-1 **Corresponding Angles Postulate**
If two parallel lines are cut by a transversal, then corresponding angles are congruent.



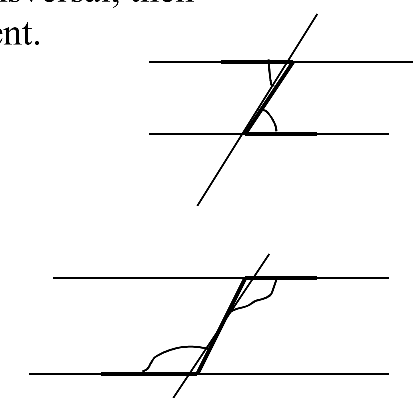
a = a
b = b
c = c
d = d



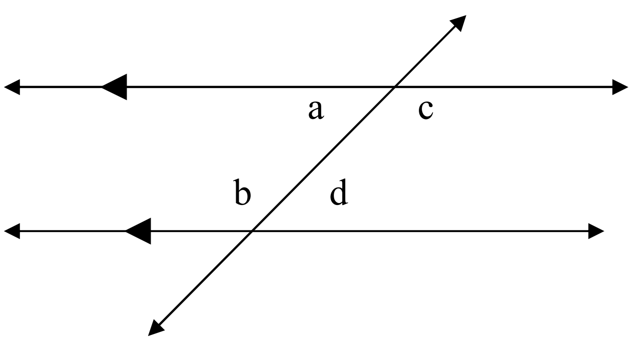
Theorem 3-1 **Alternate Interior Angles Theorem**
If two parallel lines are cut by a transversal, then alternate interior angles are congruent.



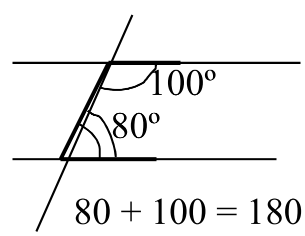
a = a
b = b



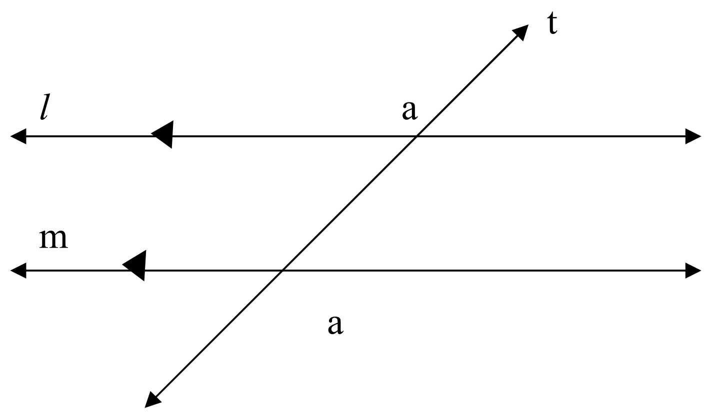
Theorem 3-2 **Same-side Interior Angles Theorem**
If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.



a + b = 180
c + d = 180

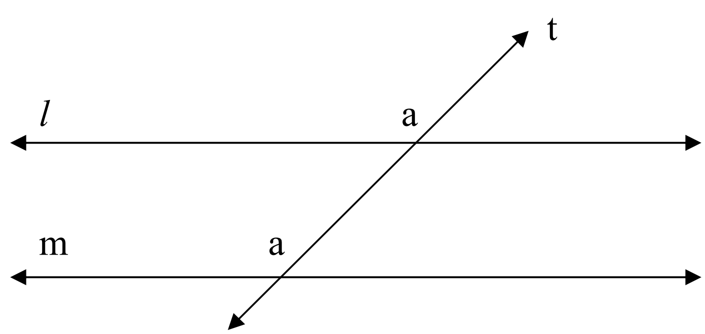


Theorem 3-3 **Alternate Exterior Angles Theorem**
If a transversal intersects two parallel lines, then alternate exterior angles are congruent.



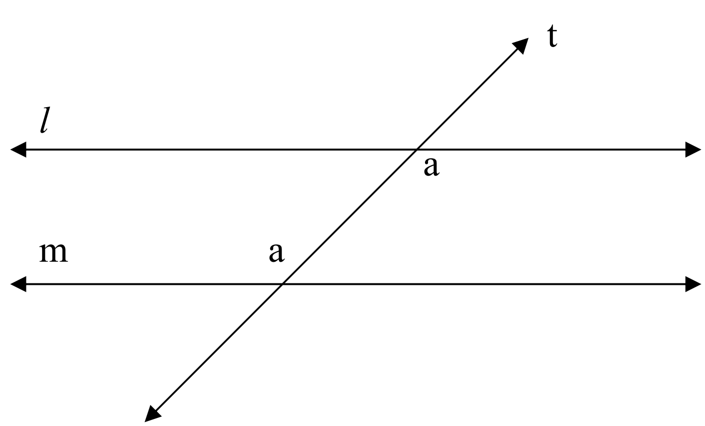
If *l* ∥ m, then a = a.

Postulate 3-2 **Converse of Corresponding Angles Postulate**
If two lines are cut by a transversal and corresponding angles are congruent, then the lines are parallel.



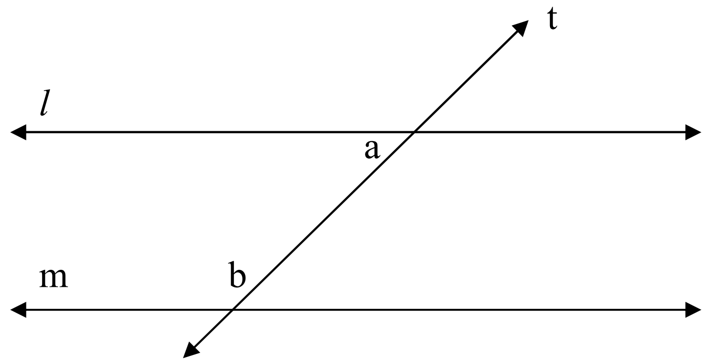
If a = a, then *l* ∥ m

Theorem 3-4 **Converse of the Alternate Interior Angles Theorem**
If two lines are cut by a transversal and alternate interior angles are congruent, then the lines are parallel.



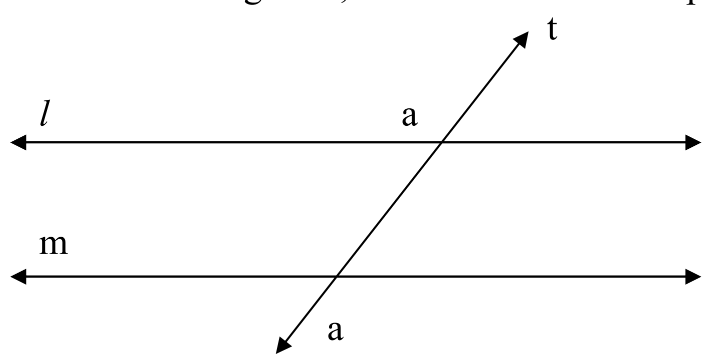
If a = a
Then lines *l* and m are parallel

Theorem 3-5 **Converse of the Same-Side Interior Angles Theorem**
If two lines are cut by a transversal and same-side interior angles are supplementary, then the lines are parallel.



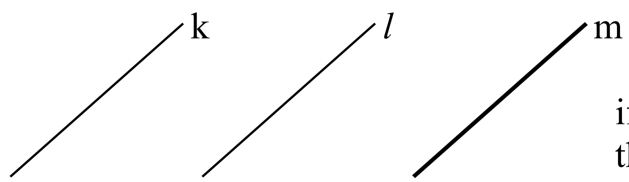
If a and b are supplementary, Then lines *l* and m are parallel.

Theorem 3-6 **Converse of the Alternate Exterior Angles Theorem**
If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.



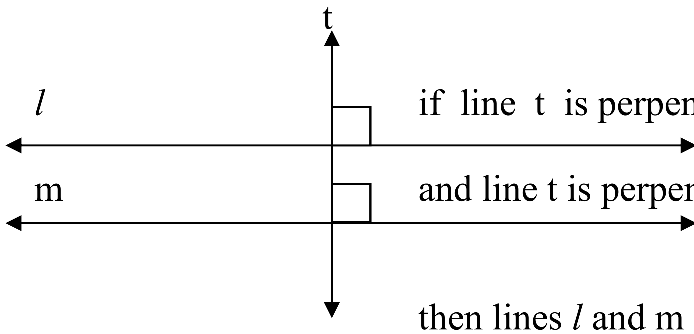
If *l* ∥ m, then a = a.

Theorem 3-7 **Two lines parallel to a third line are parallel to each other.**



if k || m and *l* || m
then k || *l*

Theorem 3-8 **In a plane two lines perpendicular to the same line are parallel.**

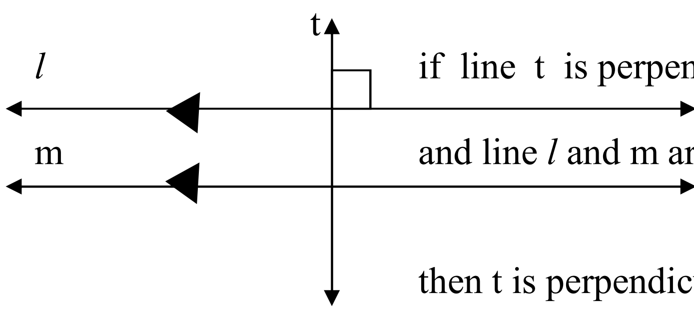


if line t is perpendicular to line *l*

and line t is perpendicular to line m

then lines *l* and m are parallel.

Theorem 3-9 **Perpendicular Transversal Theorem**
In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

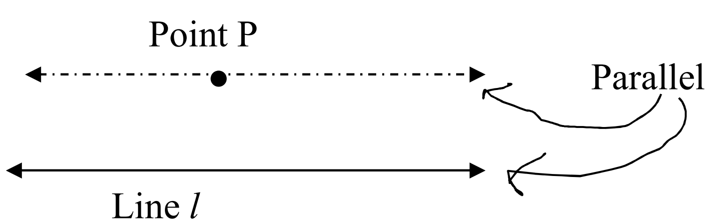


if line t is perpendicular to line *l*

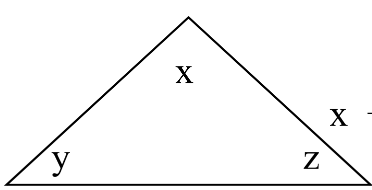
and line *l* and m are parallel

then t is perpendicular to line m

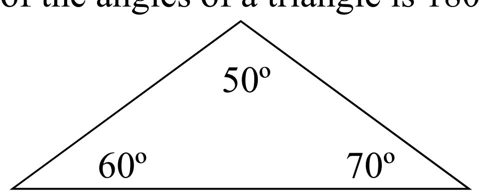
Postulate 3-3 **Parallel Postulate**
Through a point outside a line, there is exactly one line parallel to the given line.



Theorem 3-10 **Triangle Angle Sum Theorem**
The sum of the measures of the angles of a triangle is 180.

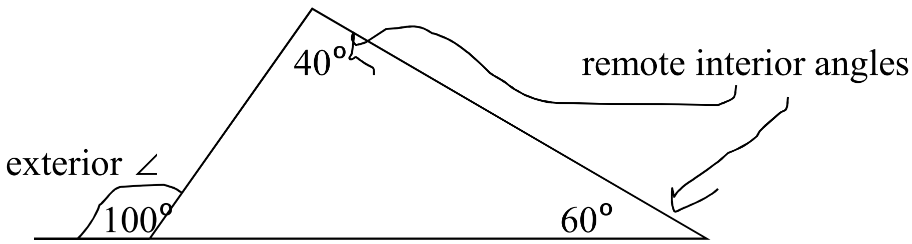


$x + y + z = 180$ or

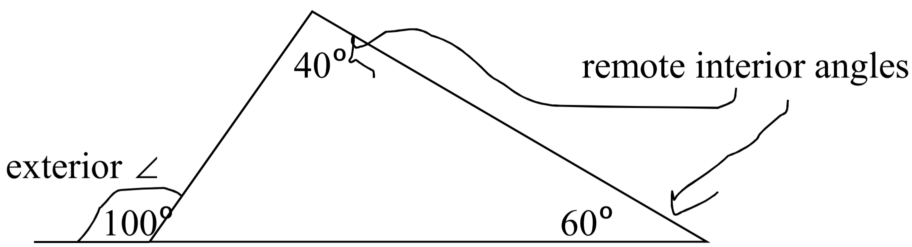


$50 + 60 + 70 = 180^\circ$

Theorem 3-11 **Triangle Exterior Angle Theorem**
The measure of an exterior angle of a triangle equals the sum of the measures of the two remote interior angles.



Corollary **(3-11)** The measure of an exterior angle of a triangle is greater than the measure of each of the two remote interior angles.

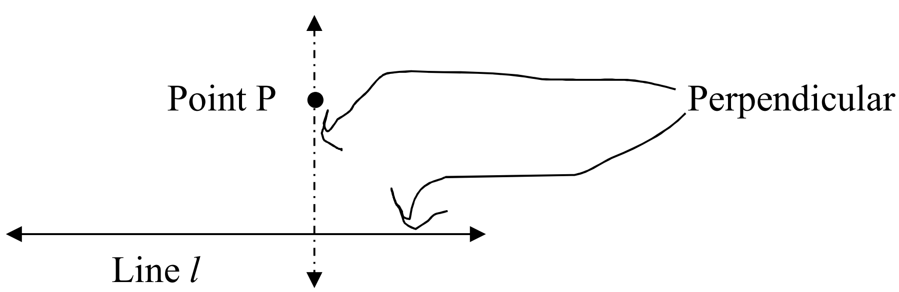


100 >40 and

100>60

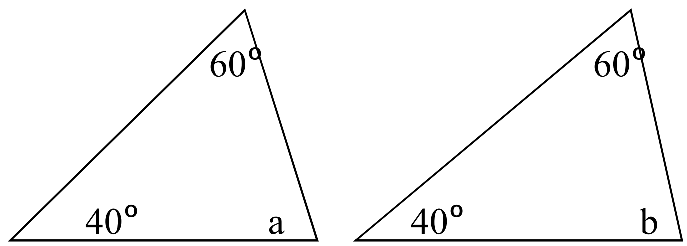
Spherical Geometry Parallel Postulate
Through a point not on a line, there is no line parallel to the given line.

Postulate 3-4 **Perpendicular Postulate**
Through a point outside a line, there is exactly one line perpendicular to the given line.



Theorem 4-1 **Third Angles Theorem**

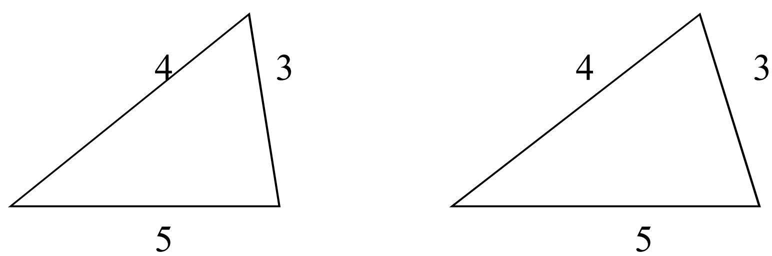
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.



if 40° = 40°
and 60° = 60°
then a = b

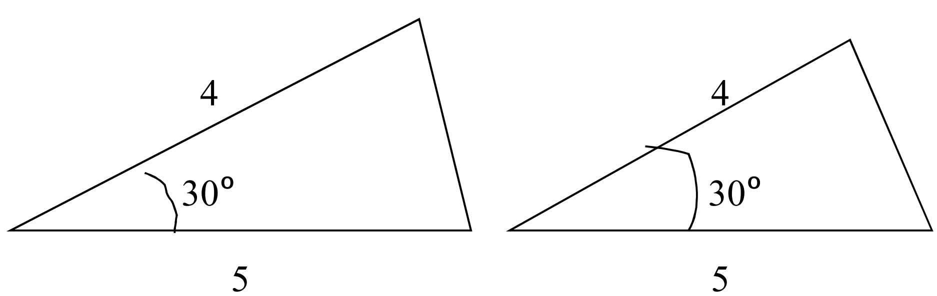
Postulate 4-1 **Side-Side-Side (SSS) Postulate.**

If three sides of one triangle are congruent to three side of another triangle, then the two triangles are congruent.

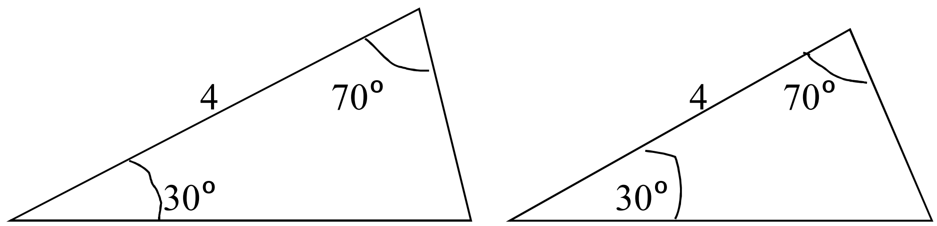


Postulate 4-2 **Side-Angle-Side (SAS) Postulate**

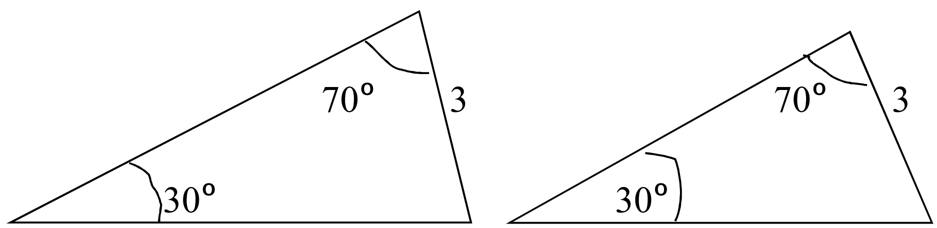
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.



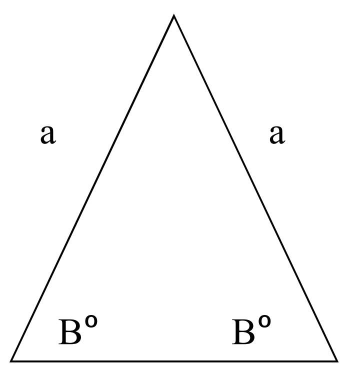
Postulate 4-3 **Angle-Side-Angle (ASA) Postulate**
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. (page 123)



Theorem 4-2 **Angle-Angle-Side (AAS) Theorem**
If two angles and a non-included side of one triangle are congruent to the corresponding parts of another triangle, then the triangles are congruent.

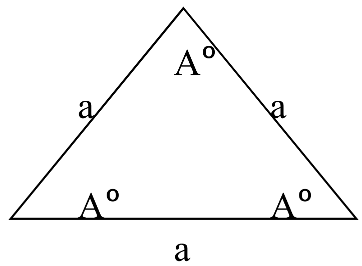


Theorem 4-3 **Isosceles Triangle Theorem**
If two sides of a triangle are congruent, then the angles opposite those sides are congruent. (page 135)



If a = a
Then B° = B°

Corollary **(4-3)** If a triangle is equilateral, then it is also equiangular.

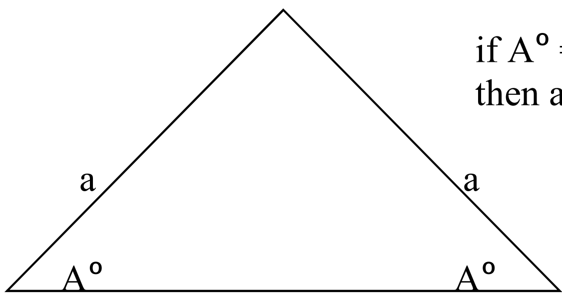


If a = a = a

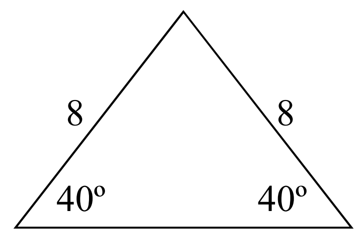
Then A° = A° = A°

Theorem 4-4 **Converse of the Isosceles Triangle Theorem**

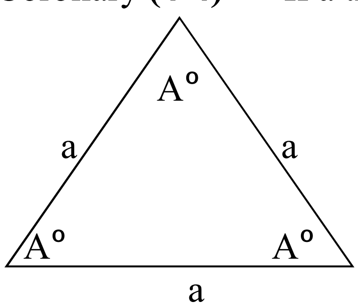
If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



if A° = A °
then a = a

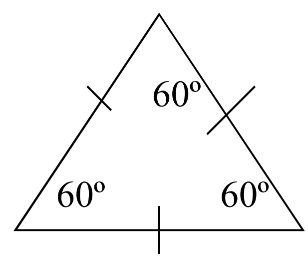


Corollary **(4-4)** If a triangle is equiangular it is also equilateral.

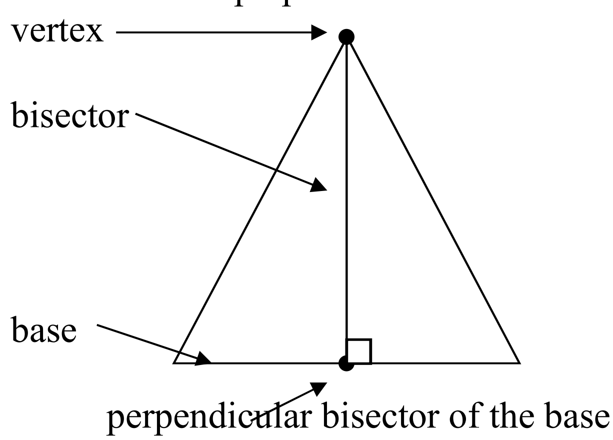


If A° = A° = A°

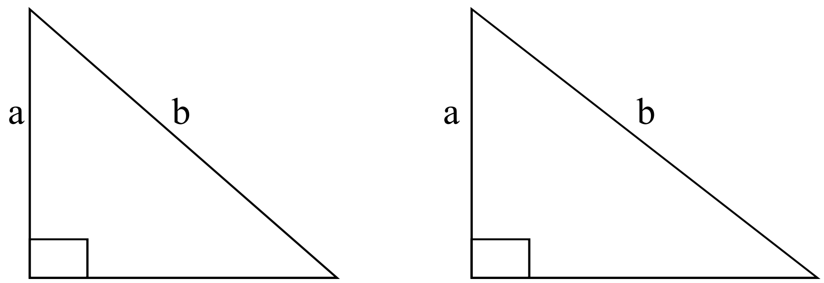
Then a = a = a



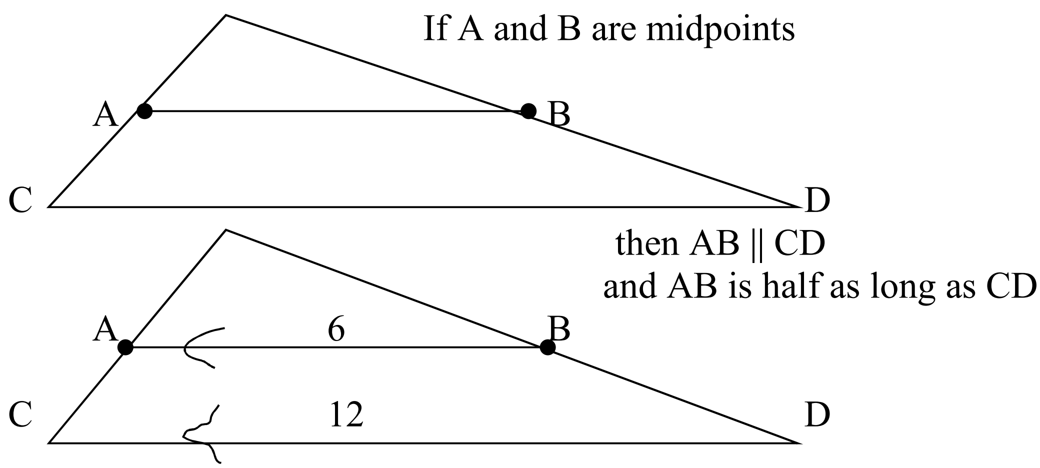
Theorem **4-5** If a line bisects the vertex angle of an isosceles triangle, then the line is also the perpendicular bisector of the base.



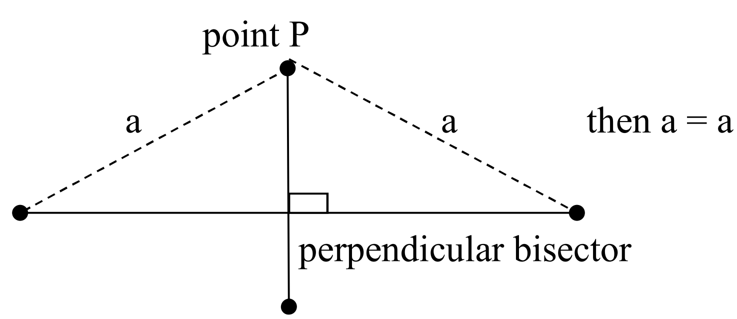
Theorem 4-6 **Hypotenuse-Leg Theorem**
If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.



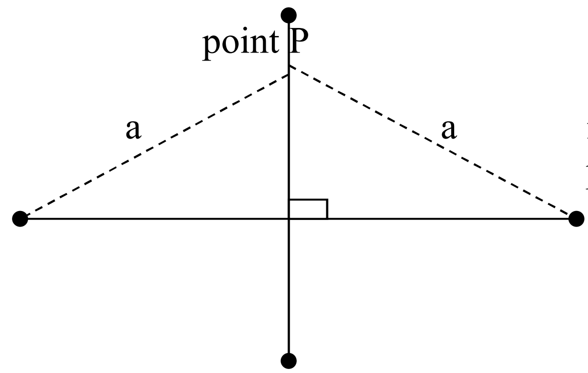
Theorem 5-1 **Triangle Mid-segment Theorem**
The segment that joins the midpoints of two sides of a triangle
(1) is parallel to the third side.
(2) Is half as long as the third side.



Theorem 5-2 **Perpendicular Bisector Theorem**
If a point lies on the perpendicular bisector of a segment , then the point is equidistant from the endpoints of the segment.

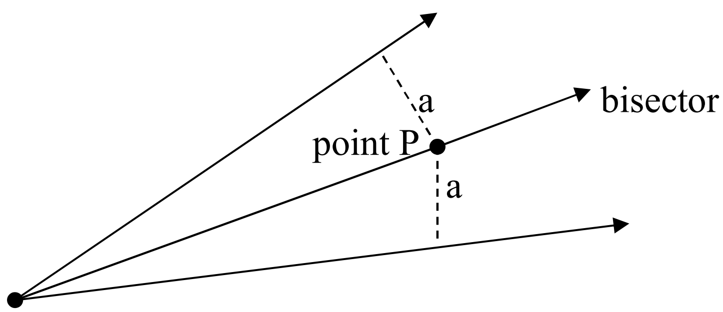


Theorem 5-3 **Converse of the Perpendicular Bisector Theorem**
If a point is equidistant from the endpoints of a segment, then the point lies on the perpendicular bisector of the segment.



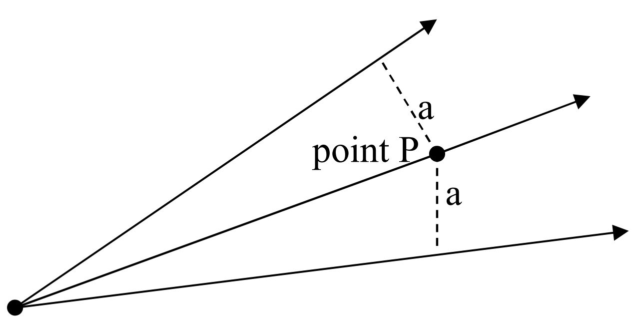
if a = a then P is on perpendicular bisector

Theorem 5-4 **Angle Bisector Theorem**
If a point lies on the bisector of an angle, then the point is equidistant from the sides of the angle.



then a = a

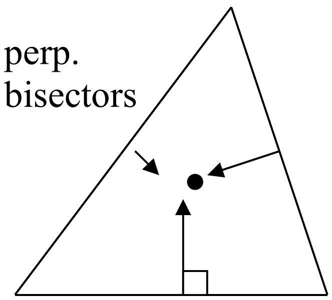
Theorem 5-5 **Converse of the Angle Bisector Theorem**
If a point is equidistant from the sides of an angle, then the point lies on the bisector of the angle.

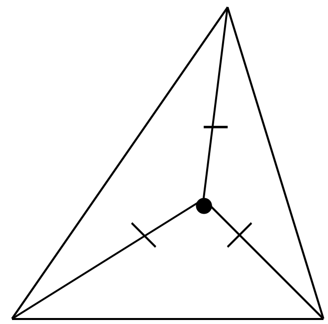


if a = a
then P is on bisector

Theorem 5-6 **Concurrency of Perpendicular Bisectors Theorem**
The perpendicular bisectors of the sides of a triangle intersect in a point that is equidistant from the three vertices of the triangle.

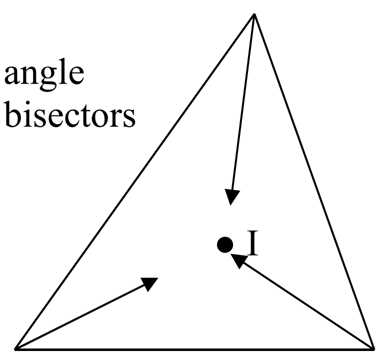
perp.
bisectors

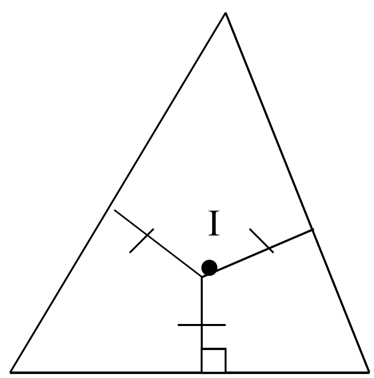




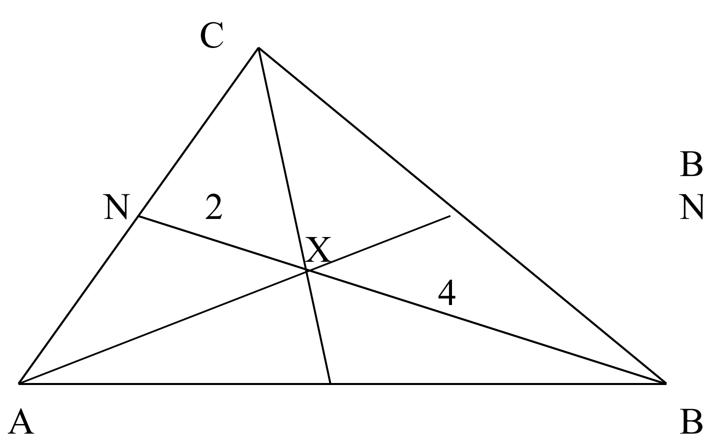
Theorem 5-7 **Concurrency of Angle Bisectors Theorem**
The bisectors of the angles of a triangle intersect in a point that is equidistant from the three sides of the triangle.

angle
bisectors



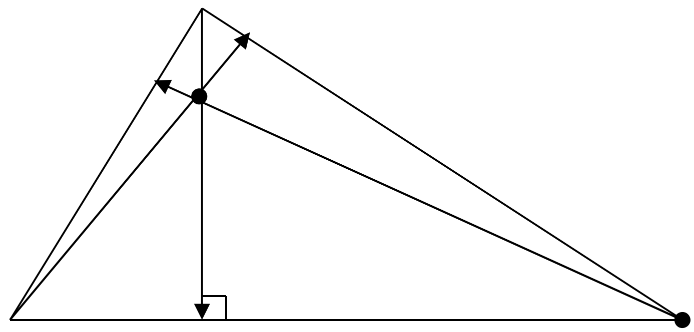


Theorem 5-8 **Concurrency of Medians Theorem**
The medians of a triangle intersect in a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

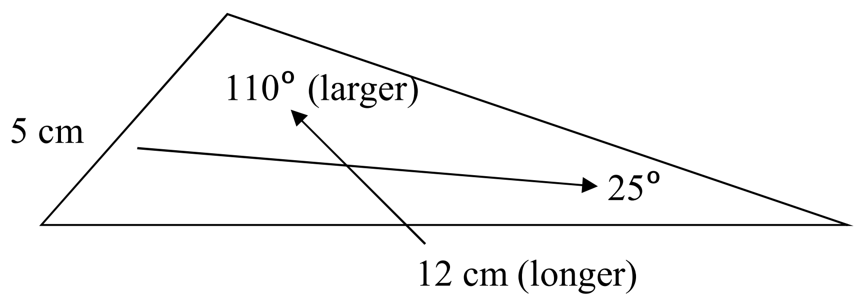


BX = 2/3 BN
NX = 1/3 NB

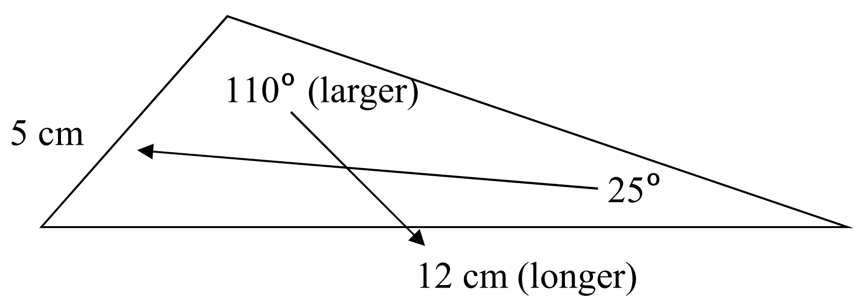
Theorem 5-9 **Concurrency of Altitudes Theorem**
The lines that contain the altitudes of a triangle intersect in a point.



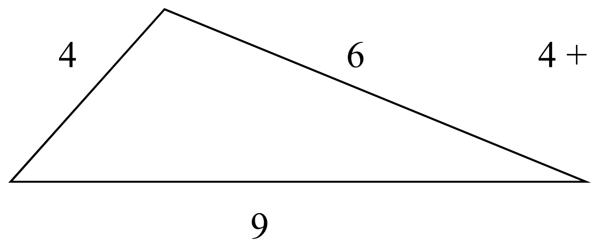
Theorem **5-10** If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.



Theorem **5-11** If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.

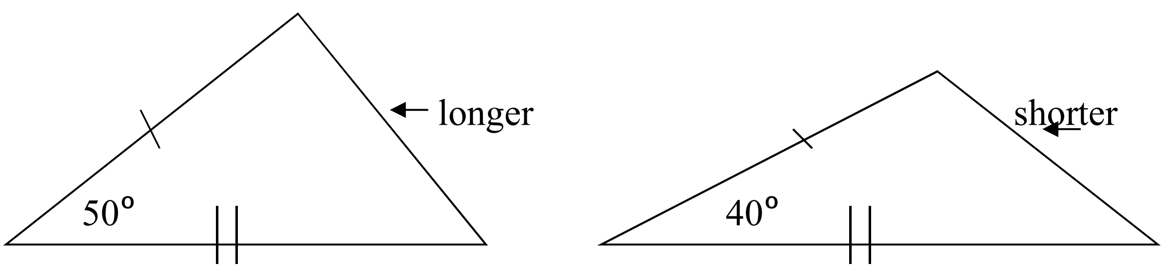


Theorem 5-12 **Triangle Inequality Theorem**
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

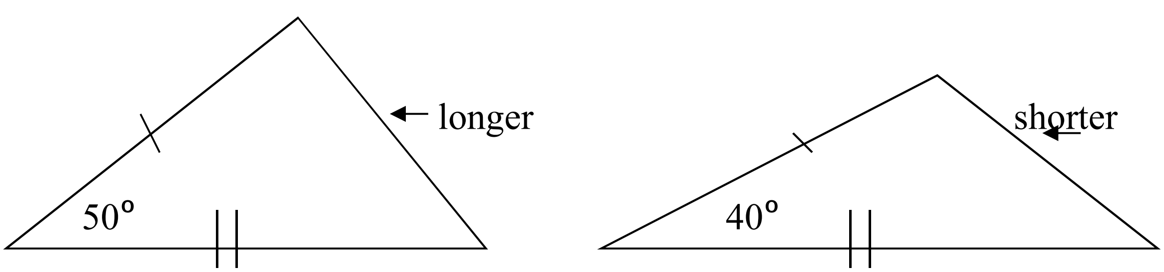


$4 + 6 > 9$

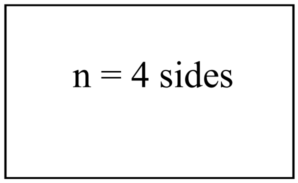
Theorem 5-13 **The Hinge Theorem (SAS Inequality Theorem)**
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is larger than the included angle of the second, then the third side of the first triangle is longer than the third side in the second triangle.



Theorem 5-14 **Converse of the Hinge Theorem (SSS Inequality)**
If two sides of one triangle are congruent to two sides of another triangle, but the third side in the first triangle is longer than the third side in the second, then the included angle of the first triangle is larger than the included angle of the second.



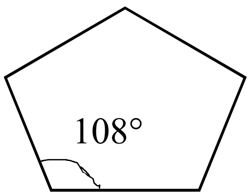
Theorem 6-1 **Polygon Angle-Sum Theorem**
The sum of the measures of the angles of a convex polygon with n sides is (n - 2)180 . (page 102)



n is the number of sides of the polygon

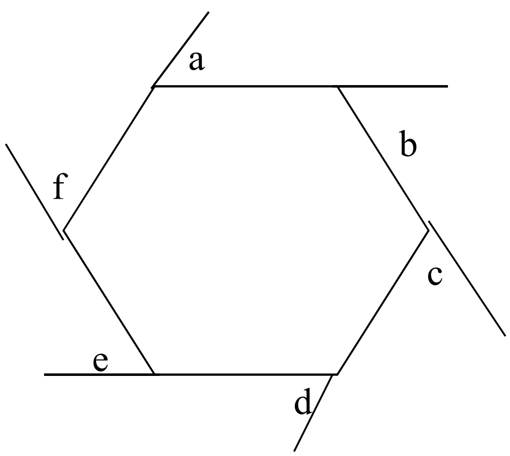
= (n - 2)180
= (4 – 2)180 = (2)180 = 360°

Corollary **(6-1)** The measure of each angle of a regular n-gon is $\frac{(n-2)180}{n}$



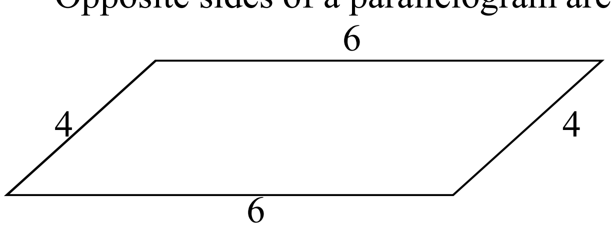
(5 – 2)180/2 = 108°

Theorem 6-2 **Polygon Exterior Angle-Sum Theorem**
The sum of the measures of the exterior angles of any convex polygon, one angle at each vertex, is 360.



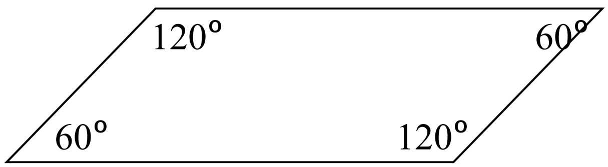
a + b + c + d + e + f = 360

Theorem **6-3** Opposite sides of a parallelogram are congruent.

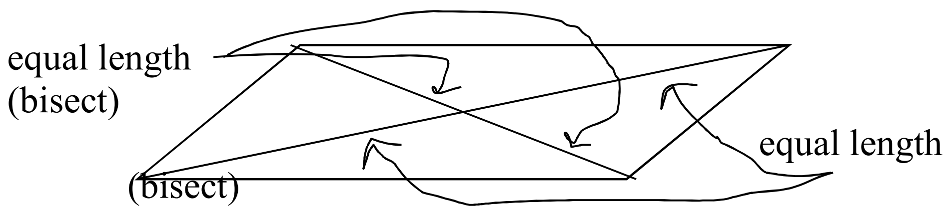


Theorem **6-4** Consecutive angles of a parallelogram are supplementary.

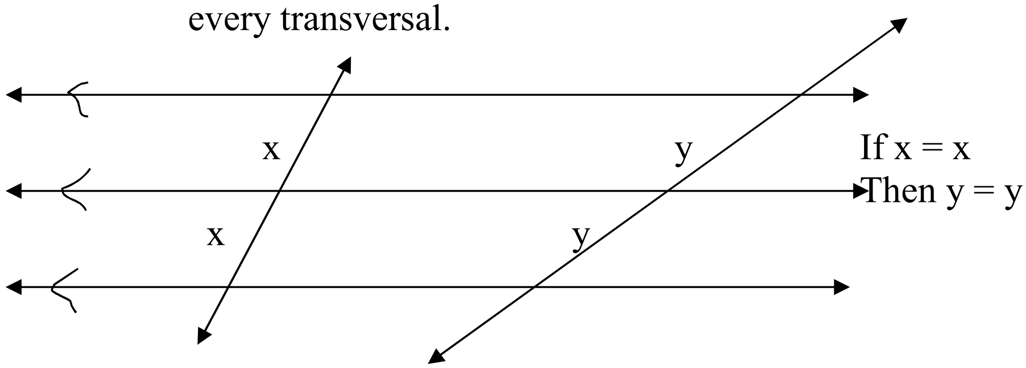
Theorem **6-5** Opposite angles of a parallelogram are congruent.



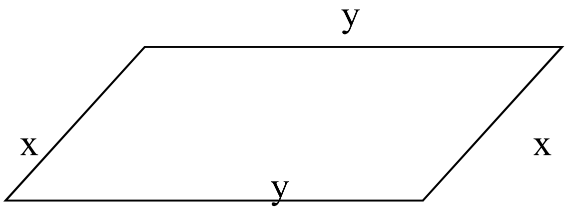
Theorem **6-6** Diagonals of a parallelogram bisect each other.



Theorem **6-7** If three parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

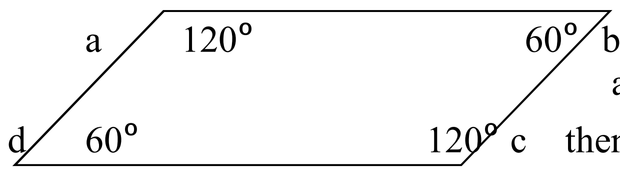


Theorem **6-8** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



If x = x and y = y
then parallelogram.

Theorem **6-9** If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.

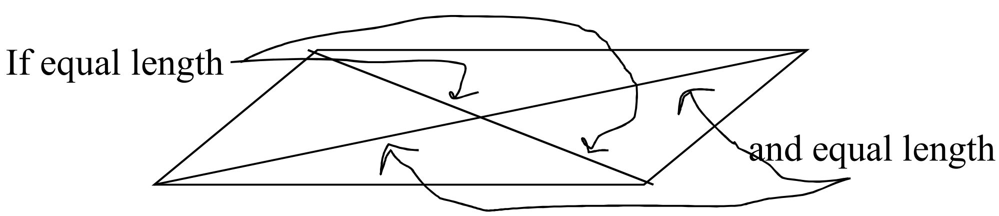


If ∠a is suppl to ∠b,
and ∠b is suppl to ∠c,
then it is a parallelogram.

Theorem **6-10** If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

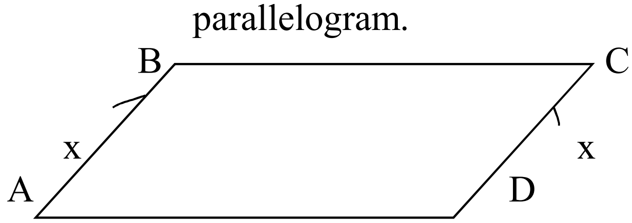
if $x^\circ = x^\circ$ and $y^\circ = y^\circ$
then parallelogram.

Theorem **6-11** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



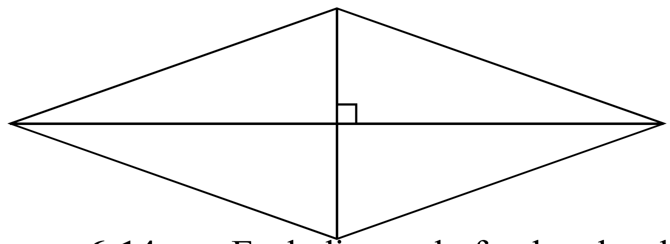
then parallelogram.

Theorem **6-12** If one pair of opposite sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.

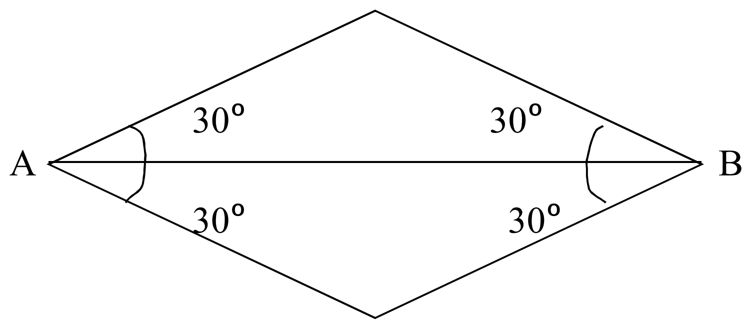


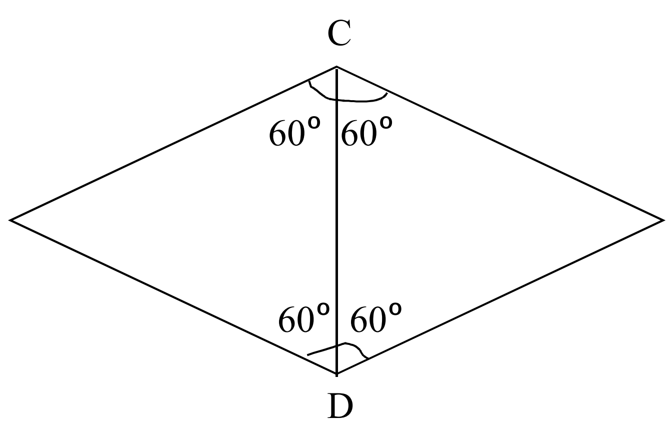
If x = x and AB || DC
then parallelogram.

Theorem **6-13** The diagonals of a rhombus are perpendicular.

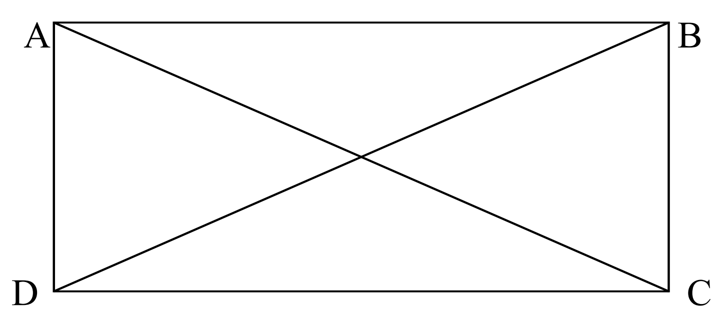


Theorem **6-14** Each diagonal of a rhombus bisects two angles of the rhombus.



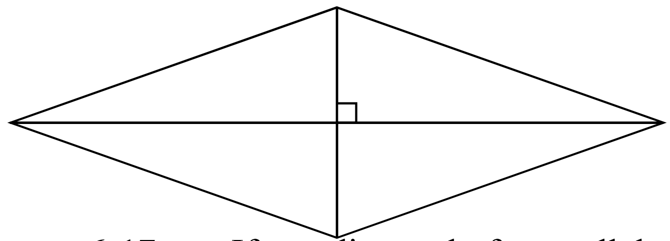


Theorem **6-15** The diagonals of a rectangle are congruent.

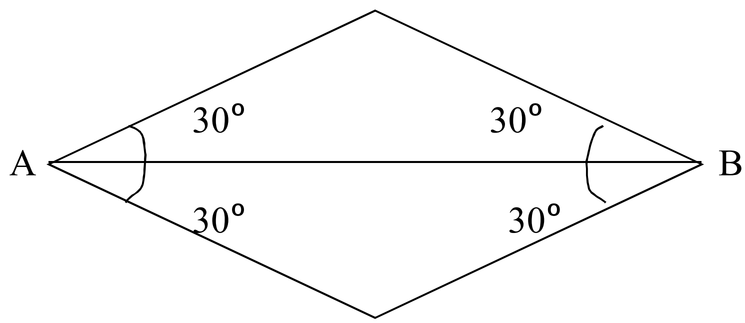


AC = BD

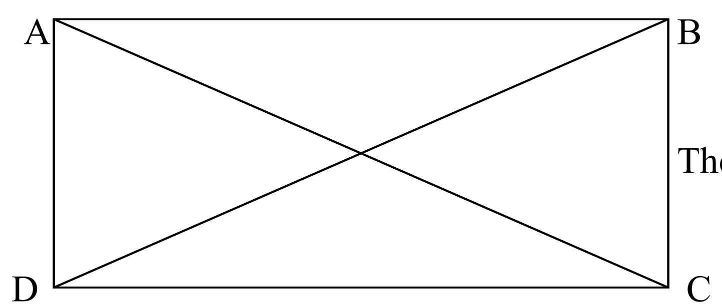
Theorem **6-16** If the diagonals of a parallelogram are perpendicular, then it is a rhombus.



Theorem **6-17** If one diagonal of a parallelogram bisects a pair of opposite angles, then it is a rhombus.

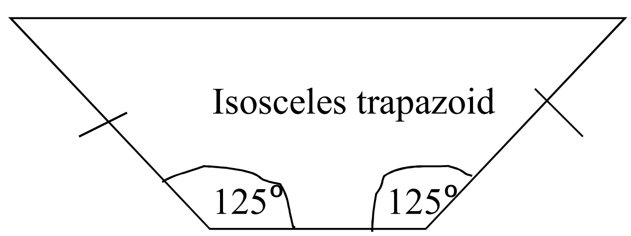


Theorem **6-18** If the diagonals of a parallelogram are congruent, then it is a rectangle.

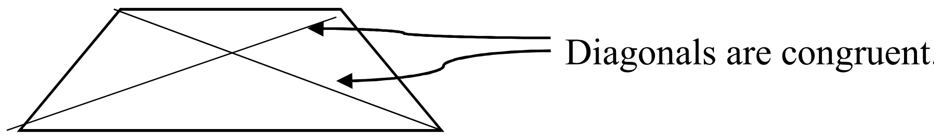


If AC = BD
Then it is a rectangle.

Theorem **6-19** Base angles of an isosceles trapezoid are congruent.



Theorem **6-20** If a quadrilateral is an isosceles trapezoid, then its diagonals are congruent.

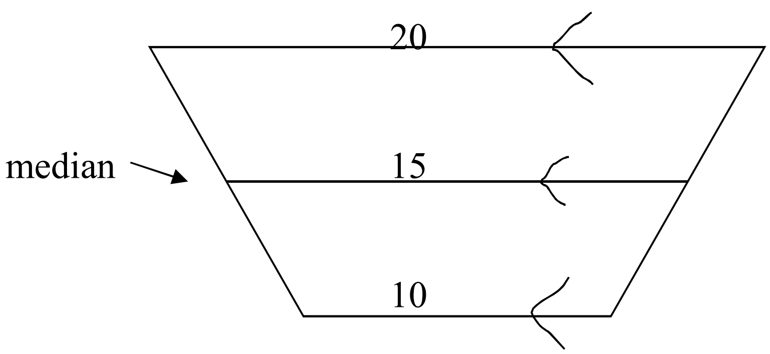


Theorem 6-21 **Trapezoid Midsegment Thoerem**

The median of a trapezoid

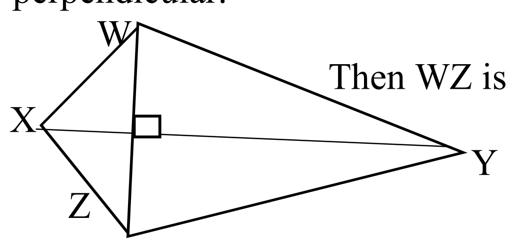
(1) is parallel to the bases

(2) has a length equal to the average of the base lengths



$$15 = \frac{20 + 10}{2}$$

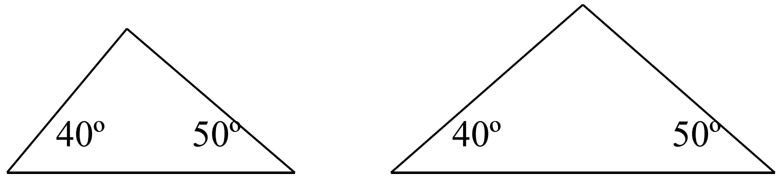
Theorem **6-22** If a quadrilateral is a kite, then its diagonals are perpendicular.



Then WZ is perpendicular to XY

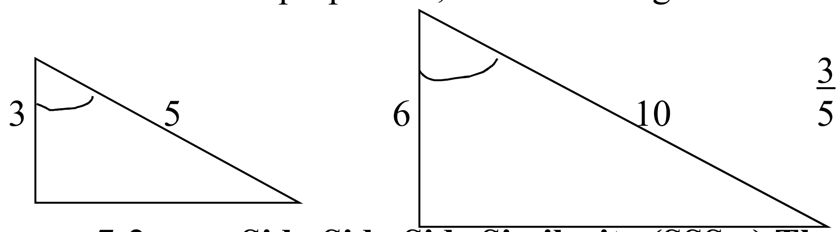
Postulate 7-1 **Angle-Angle Similarity (AA ~) Postulate**

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.



Theorem 7-1 **Side-Angle-Side Similarity (SAS ~) Theorem**

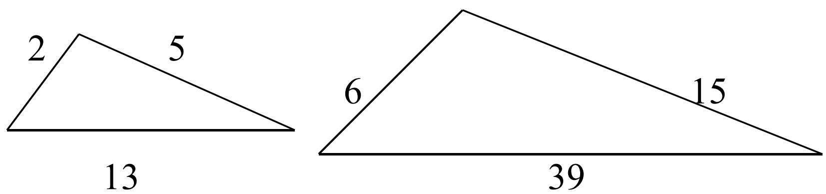
If an angle of one triangle is congruent to an angle of anothertriangle and the sides including those angles are in proportion, then the triangles are similar.



$$\frac{3}{5} = \frac{6}{10}$$

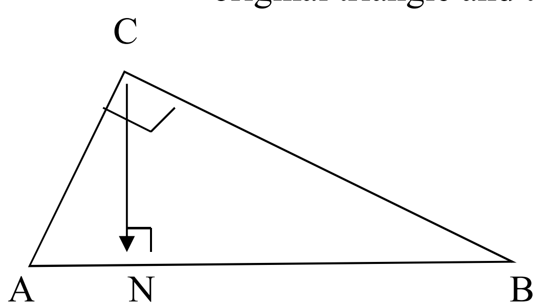
Theorem 7-2 **Side-Side-Side Similarity (SSS ~) Theorem**

If the sides of two triangles are in proportion, then the triangles are similar.



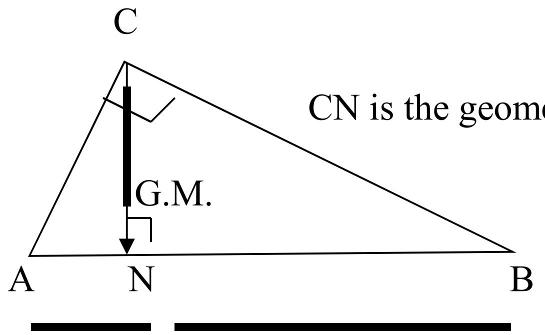
$$\frac{2}{6} = \frac{5}{15} = \frac{13}{39}$$

Theorem **7-3** If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.



$\Delta ACB \sim \Delta ANC \sim \Delta CNB$

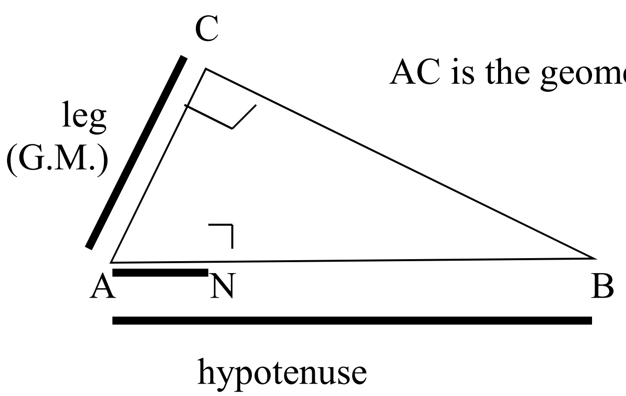
Corollary **1 (7-3)** When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean (G.M.) between the segments of the hypotenuse.



CN is the geometric mean between AN & NB

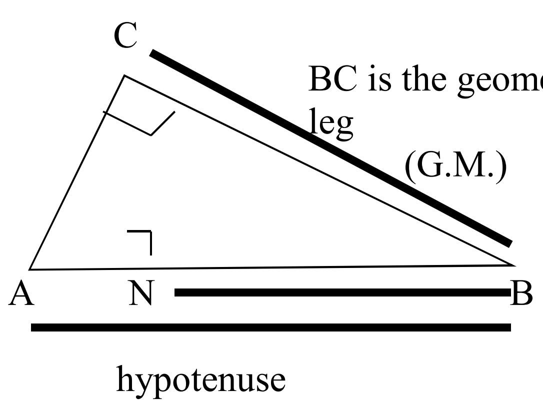
or $\frac{AN}{CN} = \frac{CN}{NB}$

Corollary **2 (7-3)** When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean (G.M.) between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.



AC is the geometric mean between AB & AN

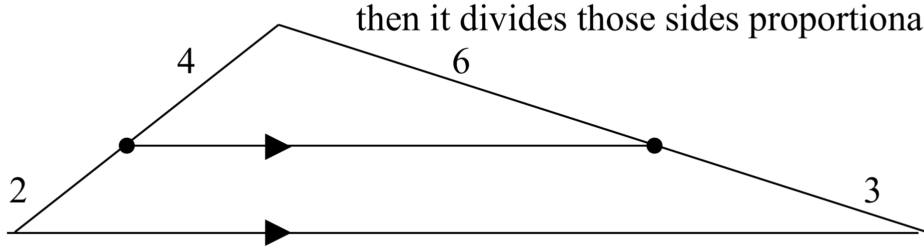
or $\frac{AB}{AC} = \frac{AC}{AN}$



BC is the geometric mean between AB & NB

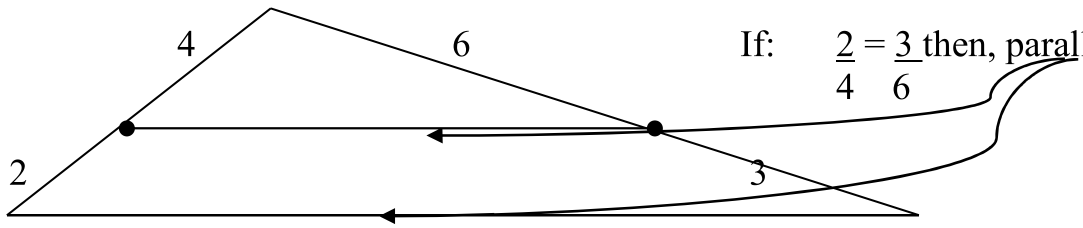
or $\frac{AB}{BC} = \frac{BC}{NB}$

Theorem 7- 4 **Side-Splitter Theorem -**If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally.



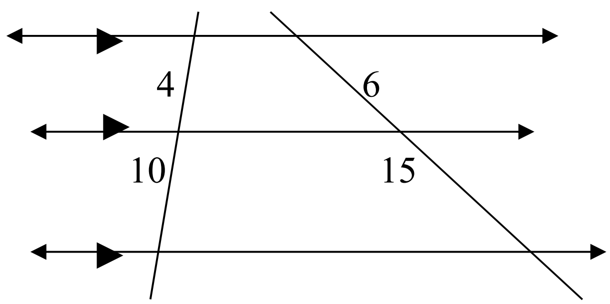
$\frac{2}{4} = \frac{3}{6}$

Converse **(7-4)** If a line divides two sides of a triangle proportionally, then it is parallel to the third side.



If: $\frac{2}{4} = \frac{3}{6}$ then, parallel

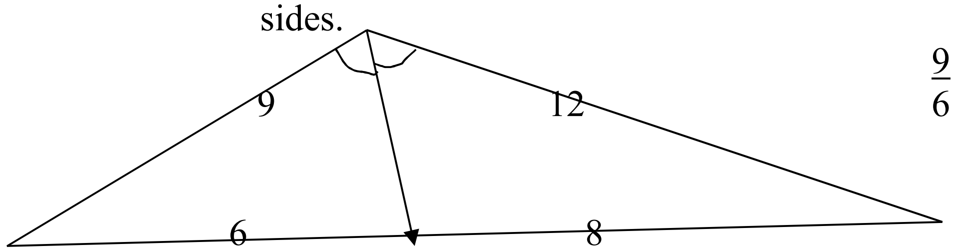
Corollary **(7-4)** If three parallel lines intersect two transversals, then they divide the transversals proportionally.



$\frac{4}{10} = \frac{6}{15}$

Theorem 7-5 **Triangle-Angle-Bisector Theorem**

If a ray bisects an angle of a triangle, then it divides the opposite side into segments proportional to the other two sides.



$\frac{9}{6} = \frac{12}{8}$

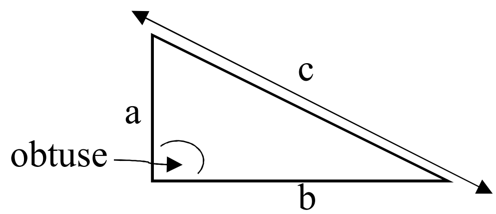
Chapter 8 – Right Triangles and Trigonometry

Note: The hypotenuse is always side c.

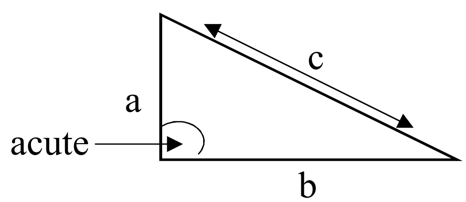
Theorem 8-1 **Pythagorean Theorem –** for a right triangle.
$a^2 + b^2 = c^2$

Theorem 8-2 **Converse of Pythagorean Theorem**
If $a^2 + b^2 = c^2$ then it is a right triangle.

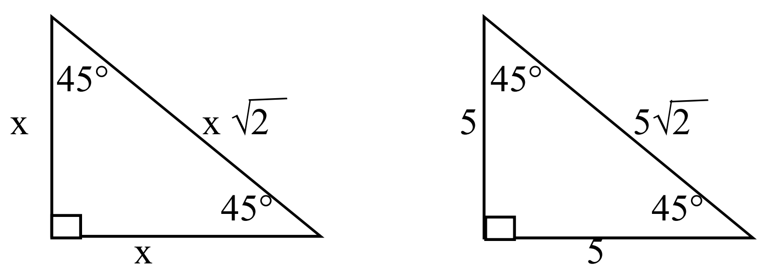
Theorem **8-3** If $c^2 > a^2 + b^2$ then it is an obtuse triangle.



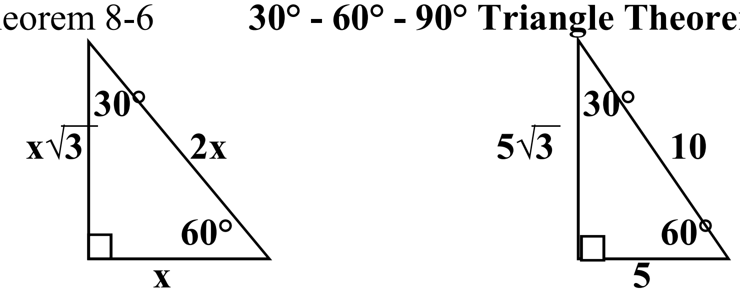
Theorem **8-4** If $c^2 < a^2 + b^2$ then it is an acute triangle.



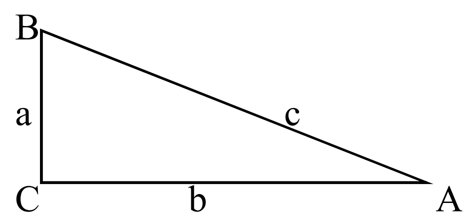
Theorem 8-5 **45° - 45° - 90° Triangle Theorem**



Theorem 8-6 **30° - 60° - 90° Triangle Theorem**



Law of Sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

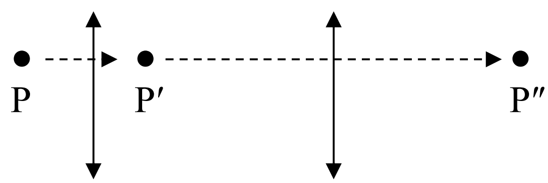


Law of Cosines $a^2 = b^2 + c^2 - 2bc \cos A$
$a^2 = b^2 + c^2 - 2bc \cos A$
$a^2 = b^2 + c^2 - 2bc \cos A$

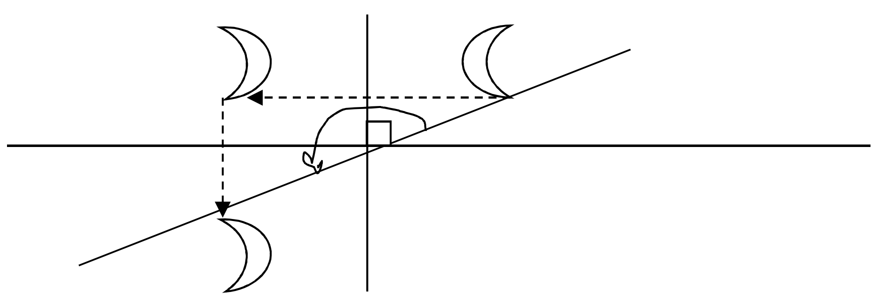
Chapter 9 - Transformations

Theorem **9-1** A translation or rotation is a composition of two reflections.

Theorem **9-2** A composite of reflections in two parallel lines is a translation.



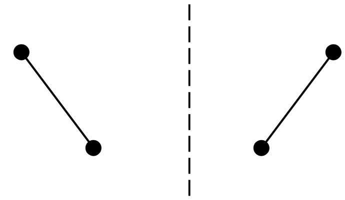
A composition of reflections across two intersecting lines is a rotation.



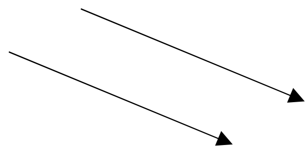
Theorem 9-3 **Fundamental Theorem of Isometries** – In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three reflections.

Theorem 9-4 **Isometry Classification Theorem** – There are only four isometries. They are translation, rotation, reflection, and glide reflection.

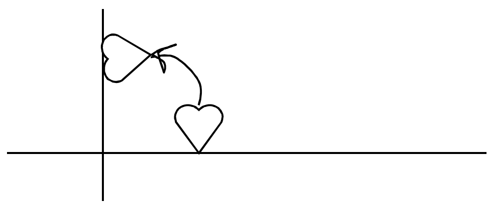
A **reflection** in a line is an isometry.



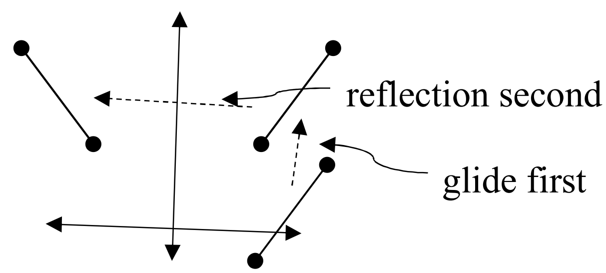
A **translation** is an isometry.



A **rotation** is an isometry.



A **glide reflection** is an isometry.



Summary of Chapter 10 Equations.

Areas:

Square	$A = s^2$
Rectangle	$A = bh$
Parallelogram	$A = bh$
Triangle	$A = \frac{1}{2} bh$
Rhombus or Kite	$A = \frac{1}{2} d_1 d_2$
Trapezoid	$A = \frac{1}{2} h(b_1 + b_2)$
Regular polygon	$A = \frac{1}{2} ap$ where a is apothem, and p is perimeter

Formulas related to circles

$C = 2\pi r$

$C = \pi d$

$A = \pi r^2$

Length of arc $\widehat{AB} = \frac{x}{360} \cdot 2\pi r$

Area of sector AOB $= \frac{x}{360} \cdot \pi r^2$

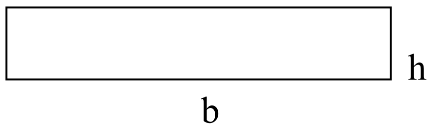
Scale Factors

Scale factor a:b

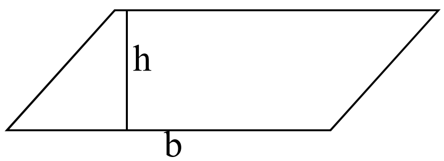
Ratio of any lengths (height, base, perimeter, etc.) a:b

Ratio of areas $a^2 : b^2$

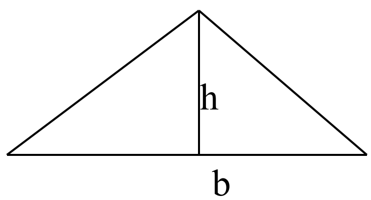
Theorem **10-1** The area of a rectangle equals the product of its base and height. ($A = bh$)



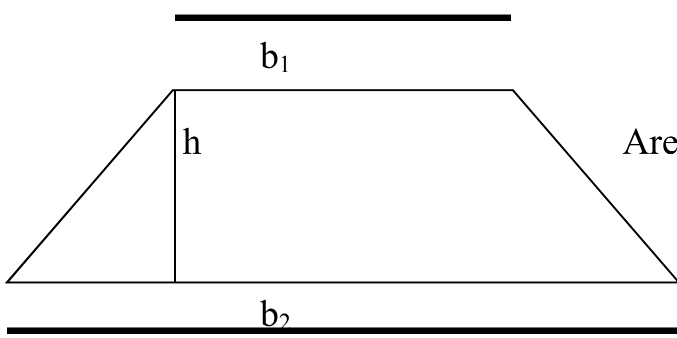
Theorem **10-2** The area of a parallelogram equals the product of a base and the height to that base. ($A = bh$)



Theorem **10-3** The area of a triangle equals half the product of a base and the height to the base. ($A = \frac{1}{2} bh$)

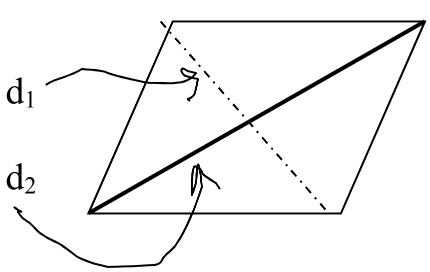


Theorem **10-4** The area of a trapezoid equals half the product of the height and the sum of the bases. ($A = \frac{1}{2} h(b_1 + b_2)$)



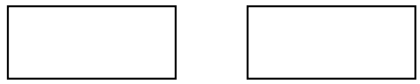
Area = $\frac{1}{2} h(b_1 + b_2)$

Theorem **10-5** The area of a rhombus or kite equals half the product of its diagonals. ($A = \frac{1}{2} d_1 d_2$)



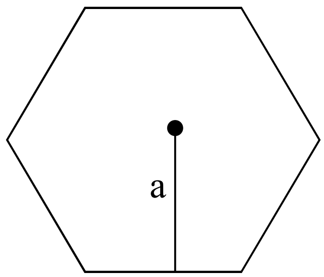
Area = $\frac{1}{2} d_1 d_2$

Postulate **10-1** If two figures are congruent, then they have the same area.



If A congruent to B
Then area of A = area of B

Theorem **10-6** The area of a regular polygon is equal to half the product of the apothem and the perimeter. (A = ½ ap)

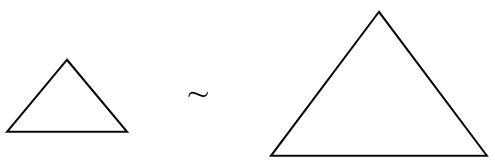


Area = ½ ap

(Perimeter is the distance all the way around the polygon.)

Theorem 10-7 **Perimeters and Areas of Similar Figures** - If the scale factor of two similar figures is a:b, then

1. the ratio of the perimeters is a:b.
2. the ratio of the areas is $a^2 : b^2$.

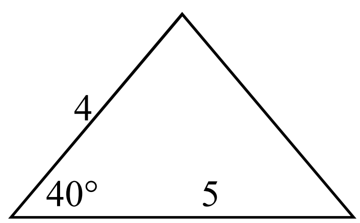


$P_1 : P_2 = a : b$
$Area_1 : Area_2 = a^2 : b^2$

scale factor is a:b

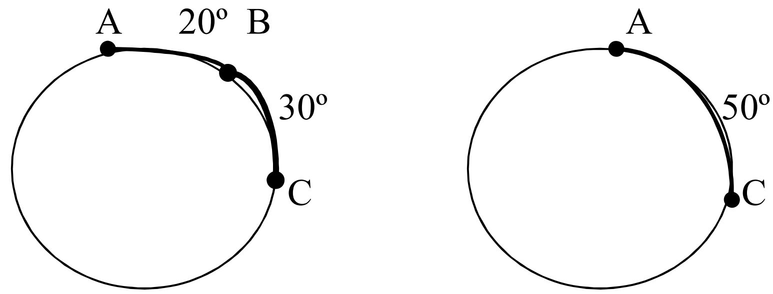
Theorem 10-8 **Area of a Triangle Given SAS** – The area of a triangle is half the product of the lengths of two sides and the sine of the included angle.
Area of ΔABC = ½bc(sin A)

Area = ½*4*5(sin 40)

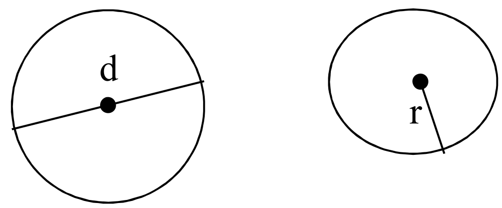


Postulate 10-2 **Arc Addition Postulate** - The measure of the arc formed by two adjacent arcs is the sum of the measures of these two arcs.

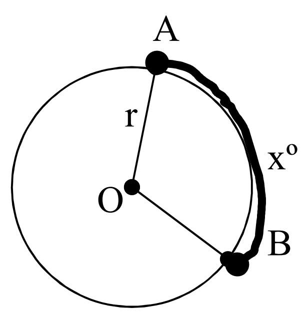
$20^o + 30^o = 50^o$



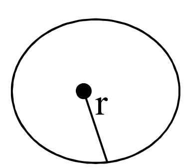
Theorem 10-9 **Circumference, C, of a circle** with diameter d: $C = \pi d$ or $C=2\pi r$



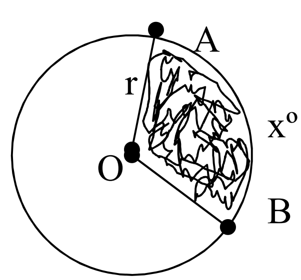
Theorem 10-10 **Arc Length** - Length of arc $\widehat{AB} = \frac{x}{360} \cdot 2\pi r$



Theorem 10-11 **Area, A, of a circle** with radius r: $A = \pi r^2$



Theorem 10-12 **Area of a sector** AOB $= \frac{x}{360} \cdot \pi r^2$



Summary of Chapter 11 Equations.

A capital "B" stands for the area of the base. An object can have one or two bases. The Lateral Area plus the area of the base/s equals the Total Area.

Solid	Lateral Area	Total area	volume
Right Prism	L.A. = ph	T.A. = L.A. + 2B	V = Bh
Pyramid	L.A. = ½ p*l*	T.A. = L.A. +B	V = 1/3 Bh
Cylinder	L.A. = ph or = $2\pi rh$	T.A. = L.A. + 2B or = L.A. + $2\pi r^2$	V = Bh or = $\pi r^2 h$
Cone	L.A. = ½ p*l* or = $\pi r l$	T.A. = L.A. + B or = L.A. + πr^2	V = 1/3 Bh or = 1/3 $\pi r^2 h$
Sphere	(not applicable)	T. A. = $4\pi r^2$	V = 4/3 πr^3

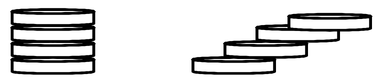
Special Right Prism Shapes

Solid	Lateral Area	Total area	volume
Right Prism (Cube)	L.A. = ph or = $4s^2$	T.A. = L.A. + 2B or = L.A. + $2s^2$	V = Bh or = s^3
Right Prism (Rectangular Solid or Square Prism)	L.A. = ph or = h(2L + 2W) Where L = length and W = width	T.A. = L.A. +2B or = L.A. + 2LW	V = Bh or = LWh

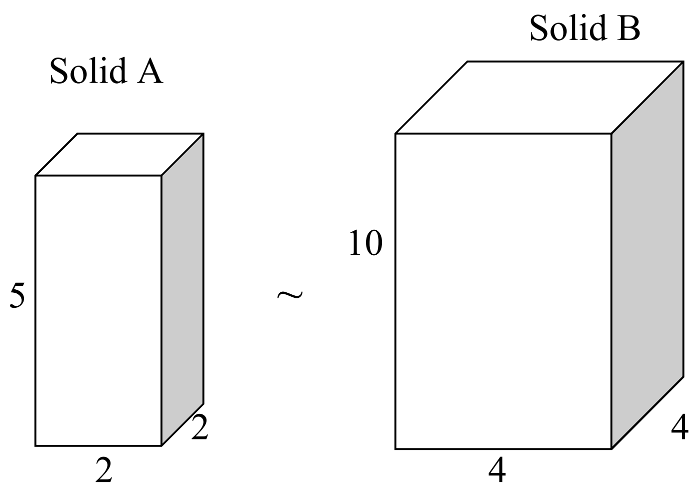
Chapter 11 Note: The pages following the Theorems show diagrams of the solids with the equations for lateral area, total area, and volume.

Note: To find the lateral area of a regular pyramid with n lateral faces:
Method 1: Find the area of one lateral face and multiply by n.
Method 2: Use the formula L.A. = ½ pl, with p = perimeter and l = slant height.

Theorem 11-5 **Cavalieri's Principle** – If two space figures have the same height and the same cross-sectional area at every level, then they have the same volume.



Theorem 11-12 **Areas and Volumes of Similar Solids** -If the scale factor of two similar solids is a:b, then
1. the ratio of corresponding perimeters is a:b
2. the ratio of the base areas, of the lateral areas, and of the total areas is $a^2 : b^2$.
3. the ratio of the volumes is $a^3 : b^3$. (page 509)

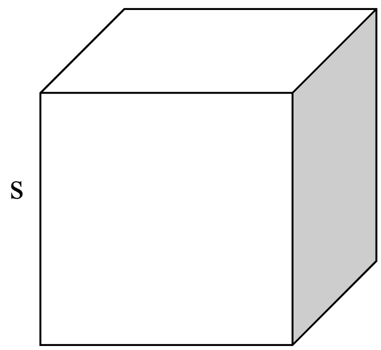


Scale factor a : b is 2 : 4 or reduced to 1:2

1. Ratio of perimeter of base is 8 : 16 or reduced to 1:2
2. Ratio of the areas of the bases is 4 : 16 or reduced to 1 : 4
this is equal to $a^2 : b^2$ or $1^2 : 2^2 = 1 : 4$
3. Ratio of volumes is 20 : 160 or reduced to 1 : 8
this is equal to $a^3 : b^3$ or $1^3 : 2^3 = 1 : 8$

These pages can be copied and used as worksheets to calculate area and volume of solids.

Cube



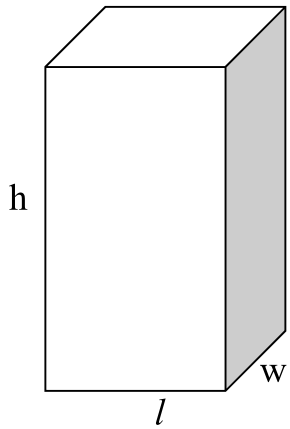
p = perimeter of the base

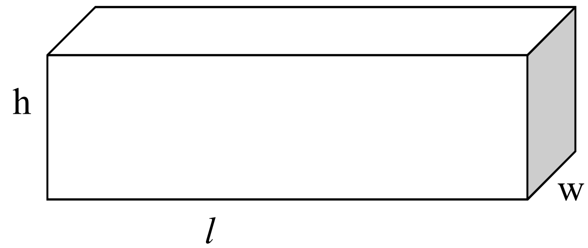
Lateral Area = ph
or
Lateral Area = $4s^2$

Volume = Bh
or
Volume = s^3

Total Area = Lateral Area + 2B
or
Total Area = Lateral Area + $2s^2$

Rectangular Solid



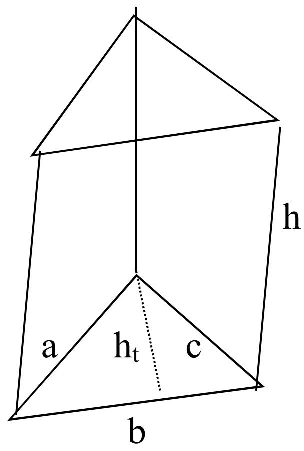


Lateral Area = ph
or
Lateral Area = $h(2l + 2w)$

Volume = Bh
or
Volume = $l \times w \times h$

Total Area = Lateral Area + 2B
or
Total Area = Lateral Area + $2 \times l \times W$

Right Triangular Prism



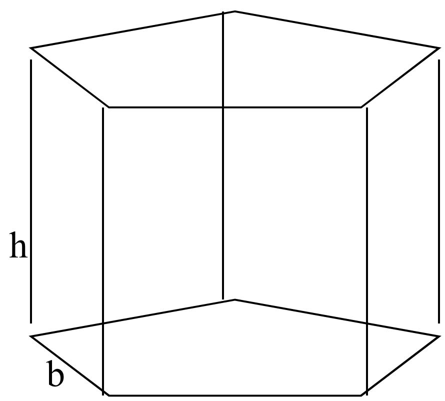
b is the base of the triangle
h_t is the height of the triangle

Lateral Area = ph
or
Lateral area = h(a + b + c)

Volume = Bh
or
Volume = ½ b×h_t×h

Total Area = Lateral Area + 2B
Or
Total area = Lateral area + 2 × ½ b×h

Right Prism

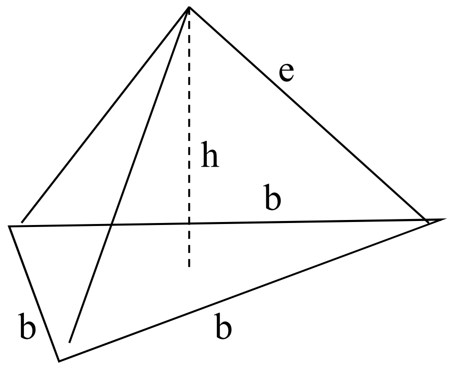


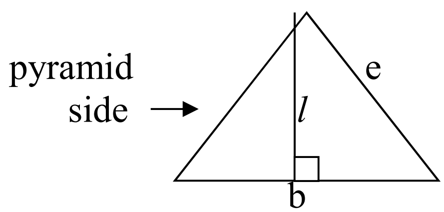
Lateral Area = ph
or
Lateral area = h(5b)

Volume = Bh

Total Area = Lateral Area + 2B

Triangular Pyramid





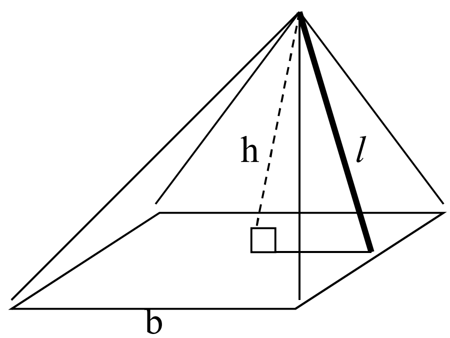
e = length of edge
l =slant height

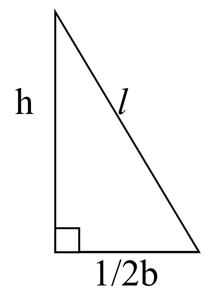
Lateral Area = ½ pl
or
Lateral Area = ½ (3b) l

Volume = 1/3 Bh

Total Area = Lateral Area + B

Square Pyramid





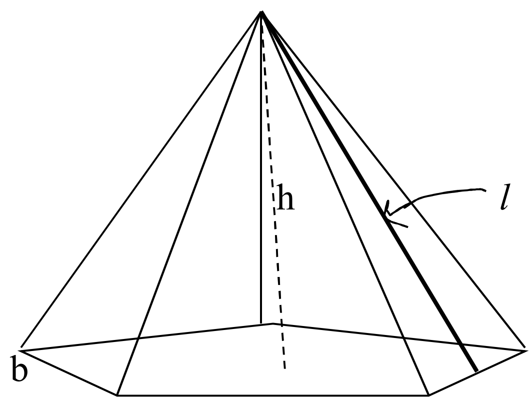
Lateral Area = ½ pl
or
Lateral Area = ½ (4b)l

Volume = 1/3 Bh

Volume = 1/3 b^2 h

Total area = Lateral Area + B
or
Total area = Lateral Area + b^2

Regular Pyramid

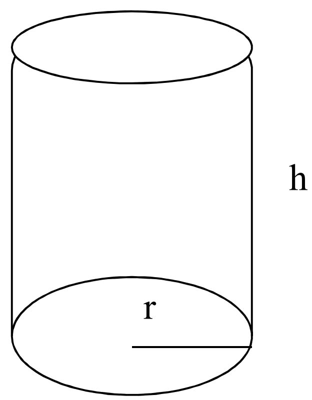


Lateral Area = ½ p*l*

Volume = 1/3 Bh

Total area = Lateral Area + B

Cylinder



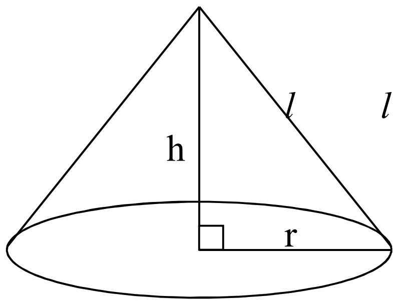
reminder: p = perimeter
C = circumference

Lateral Area = ph or Ch
or
Lateral Area = $2\pi rh$

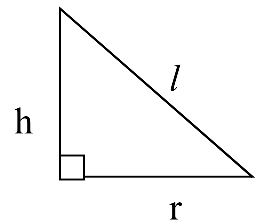
Volume = Bh
or
Volume = $\pi r^2 h$

Total area = Lateral Area + 2B
or
Total area = Lateral Area + $2\pi r^2$

Cone



l = slant height

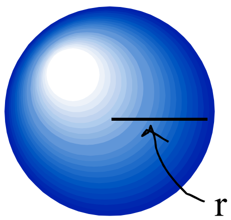


Lateral Area = ½ p*l* = ½ C*l* = ½ × 2πr × *l*
or
Lateral Area = πr*l*

Volume = 1/3 Bh
or
Volume = 1/3 π r^2h

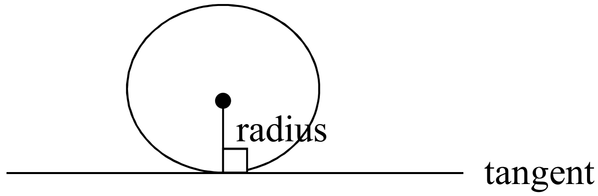
Total Area = Lateral Area + B
or
Total Area = Lateral Area + π r^2

Sphere

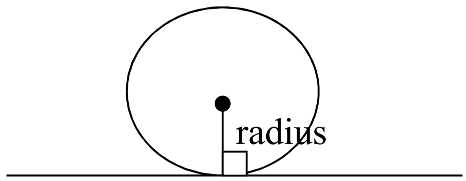


Total Area or Surface Area = $4 \pi r^2$ Volume = $4/3 \pi r^3$

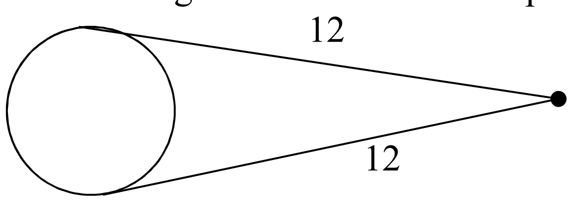
Theorem **12-1** If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.



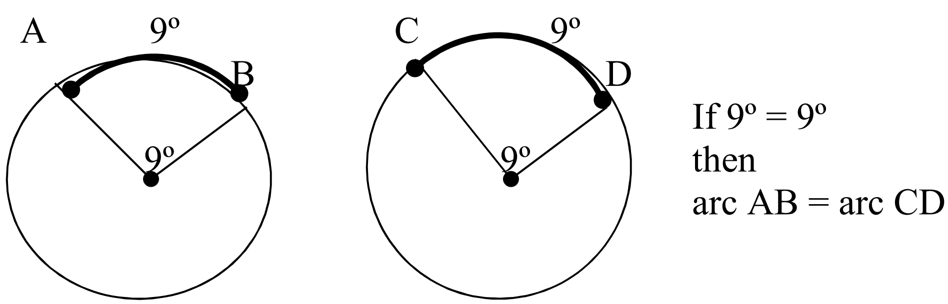
Theorem **12-2** If a line in the plane of a circle is perpendicular to a radius at its outer endpoint, then the line is tangent to the circle.



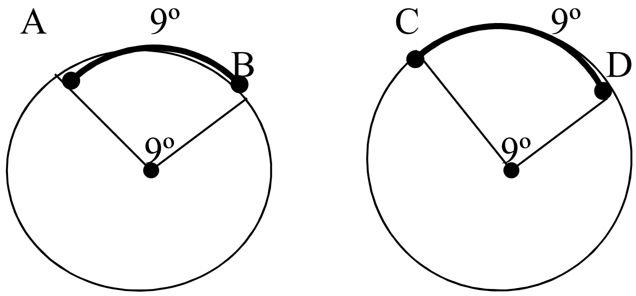
Theorem **12-3** Tangents to a circle from a point are congruent.



Theorem **12-4** In the same circle or in congruent circles, congruent central angles have congruent arcs.

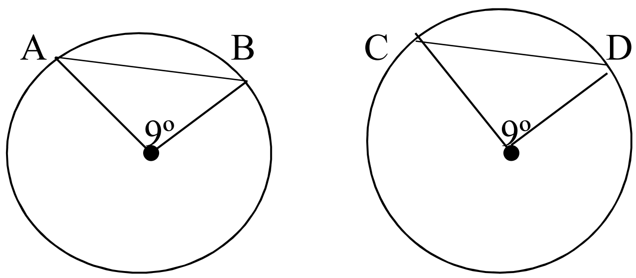


Converse (**12-4**) In the same circle or in congruent circles, congruent arcs have congruent central angles.



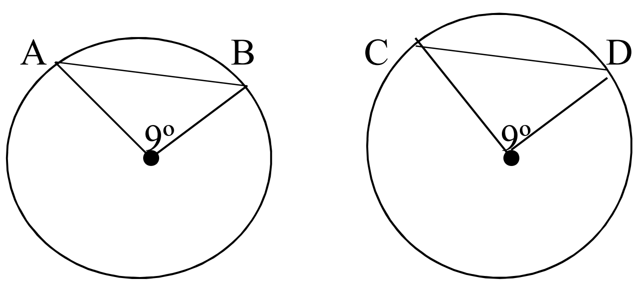
If arc AB = arc CD
then
9° = 9°

Theorem **12-5** In the same circle or in congruent circles, congruent central angles have congruent chords.



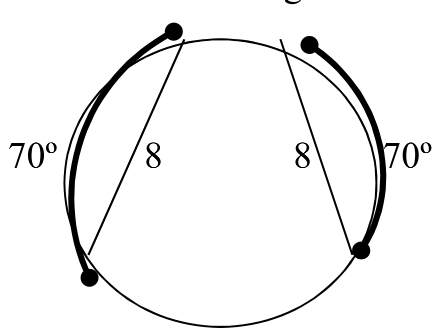
If 9º = 9º
then
AB = CD

Converse (**12-5**) In the same circle or in congruent circles, congruent chords have congruent central angles.



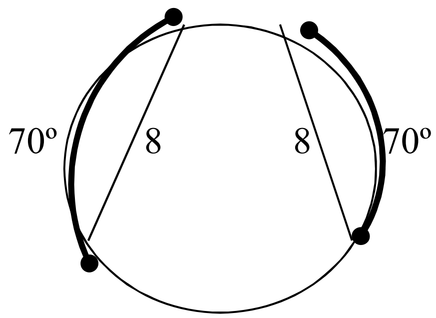
If AB = CD
then
9º = 9º

Theorem **12-6** In the same circle or in congruent circles,
Congruent chords have congruent arcs.



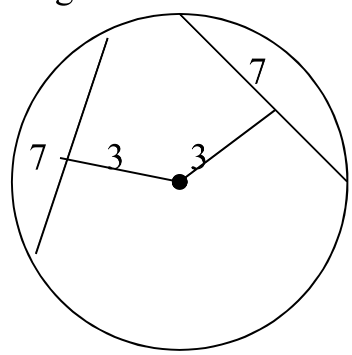
If 8 = 8,
then
70° = 70°

Converse (**12-6**) In the same circle or in congruent circles,
Congruent arcs have congruent chords.

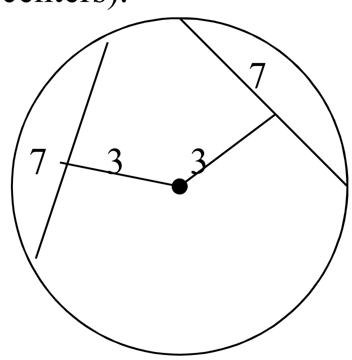


If 70° = 70°,
then
8 = 8

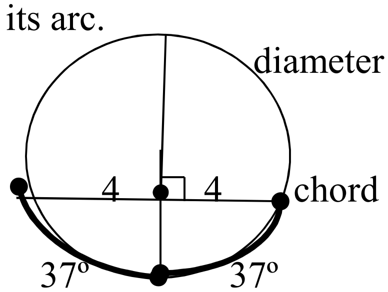
Theorem **12-7** In the same circle or in congruent circles,
Chords equally distant from the center (or centers) are congruent.



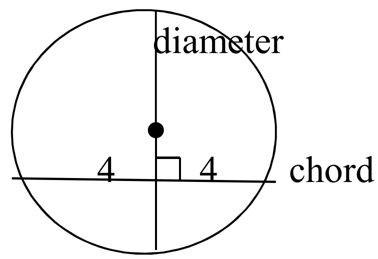
Converse (**12-7**) In the same circle or in congruent circles,
Congruent chords are equally distant from the center (or centers).



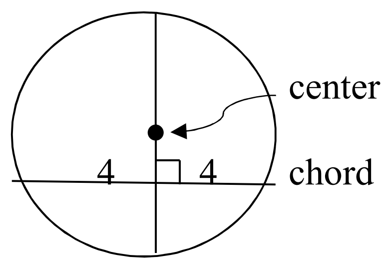
Theorem **12-8** A diameter that is perpendicular to a chord bisects the chord and its arc.



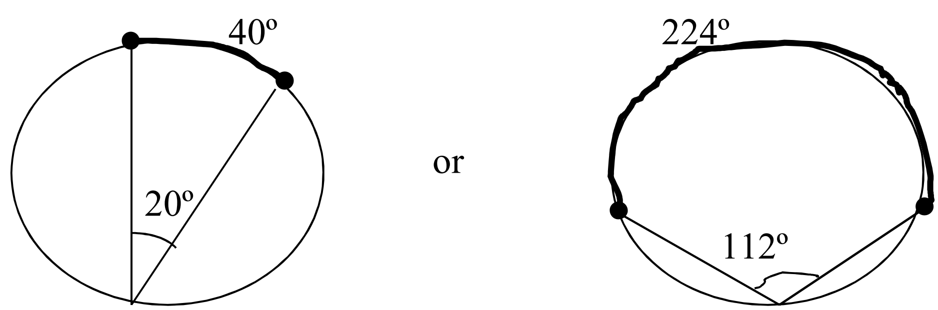
Theorem **12-9** If a diameter bisects a chord (that is not a diameter), it is perpendicular to the chord.



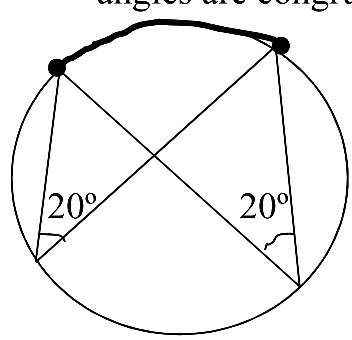
Theorem **12-10** The perpendicular bisector of a chord contains the center of the circle.



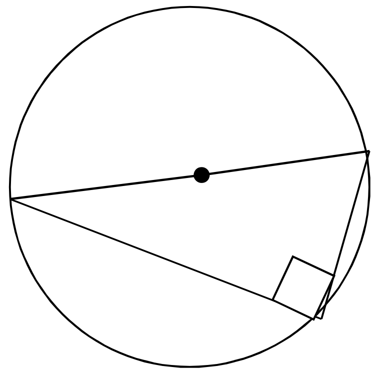
Theorem 12-11 **Inscribed Angle Theorem** - The measure of an inscribed angle is equal to half the measure of its intercepted arc.



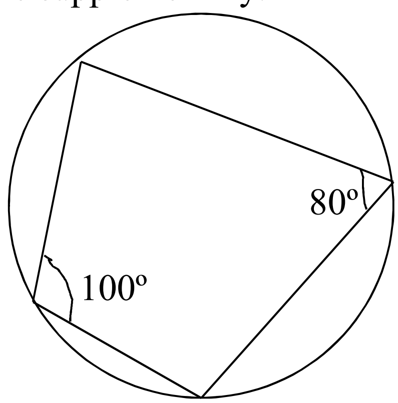
Corollary **1 (12-11)** If two inscribed angles intercept the same arc, then the angles are congruent.



Corollary 2 **(12-11)** An angle inscribed in a semicircle is a right angle.

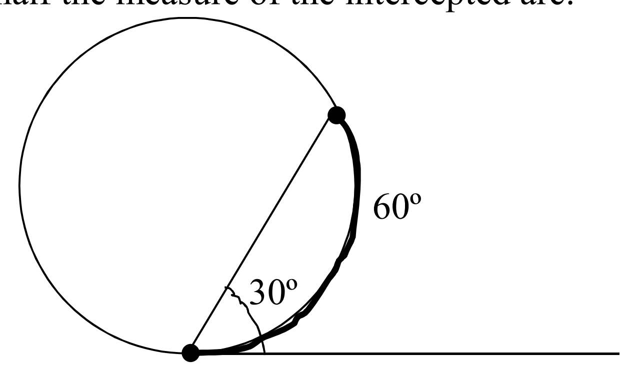


Corollary **3 (12-11)** If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

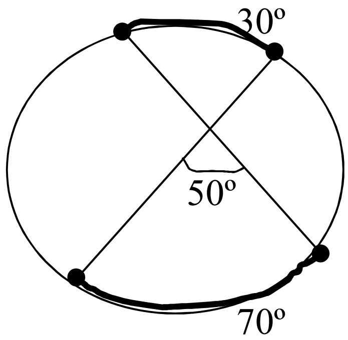


100 + 80 = 180

Theorem **12-12** The measure of an angle formed by a chord and a tangent is equal to half the measure of the intercepted arc.

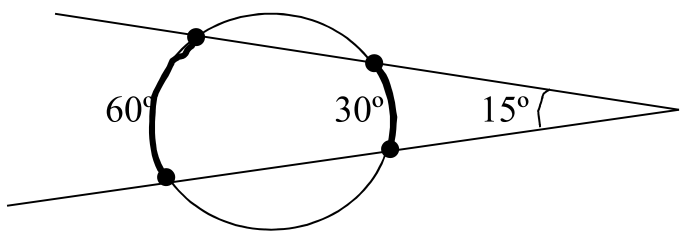


Theorem 12-13 The measure of an angle formed by two chords that intersect inside a circle is equal to half the sum of the measures of the intercepted arcs.

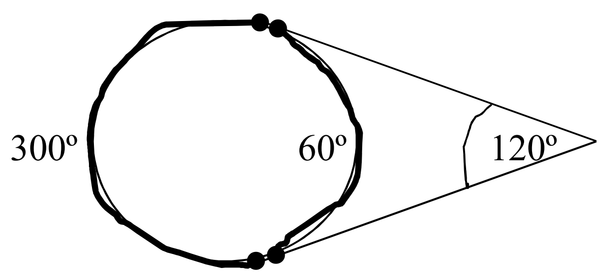


½(70 + 30) = 50º

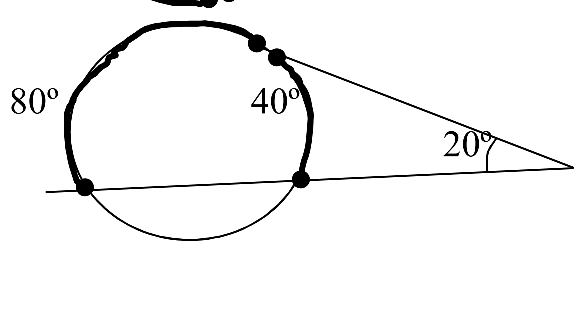
Theorem 12-14 The measure of an angle formed by two secants, two tangents, or a secant and a tangent drawn from a point outside a circle is equal to half the difference of the measures of the intercepted arcs.



½(60 – 30) = 15º



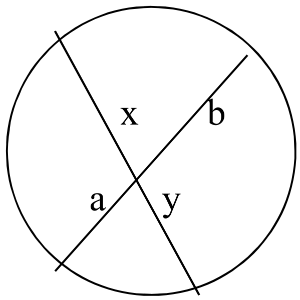
½(300 – 60) = 120º



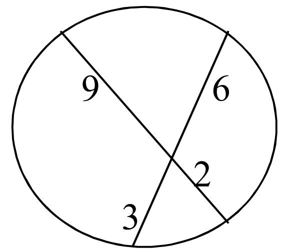
½(80 – 40) = 20º

Theorem **12-15** When two chords intersect inside a circle, the product of the segments of one chord equals the product of the segments of the other chord. (page 362)

$x \times y = a \times b$

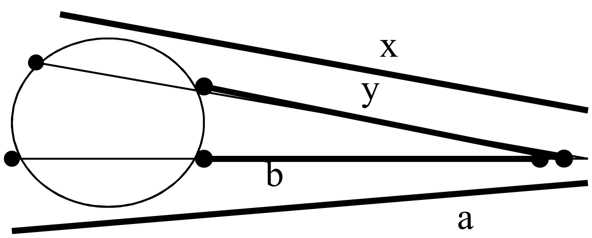


or



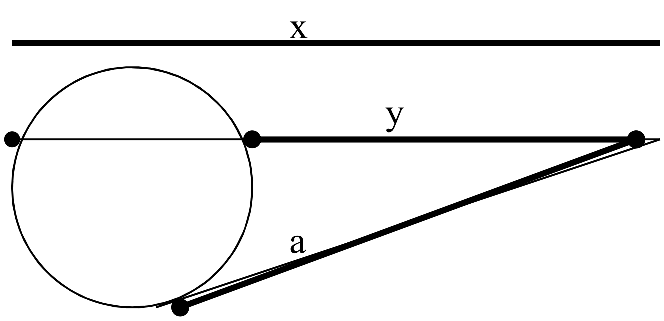
$9 \times 2 = 3 \times 6$
$18 = 18$

Theorem **12-15 also?** When two secant segments are drawn to a circle from an external point, the product of one secant segment and its external segment equals the product of the other secant segment and its external segment. (page 362)



$x \times y = a \times b$

Theorem **12-15 also?** When a secant segment and a tangent segment are drawn to a circle from an external point, the product of the secant segment and its external segment is equal to the square of the tangent segment. (page 363)



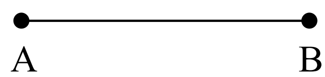
$x \times y = a \times a$

or $x \times y = a^2$

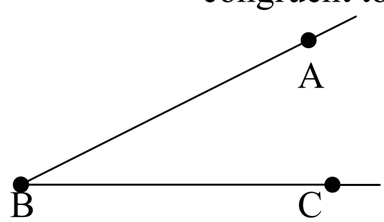
Theorem **12-16** An equation of a circle with center (h, k) and radius r is $(x – h)^2 + (y – k)^2 = r^2$

Use what is given, to practice doing the constructions.

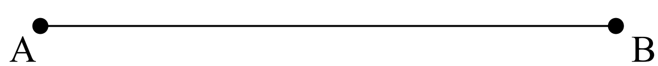
Construction 1 **Congruent Segments** -Given a segment, construct a segment congruent to the given segment.



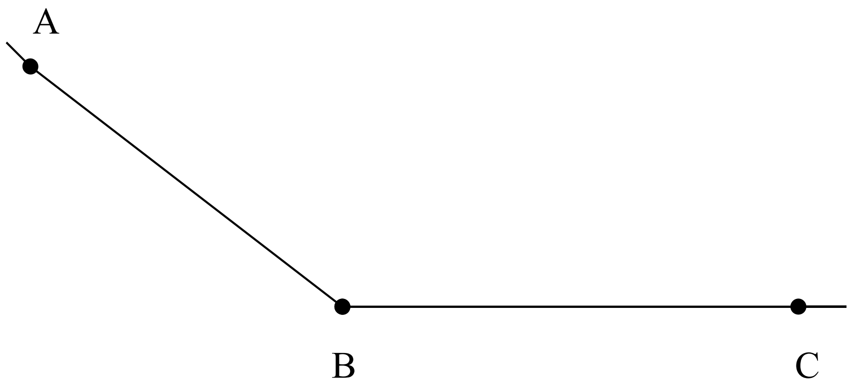
Construction 2 **Congruent Angles** - Given an angle, construct an angle congruent to the given angle.



Construction 3 **Perpendicular Bisector** - Given a segment, construct the perpendicular bisector of the segment.



Construction 4 **Angle Bisector** - Given an angle, construct the bisector of the angle.



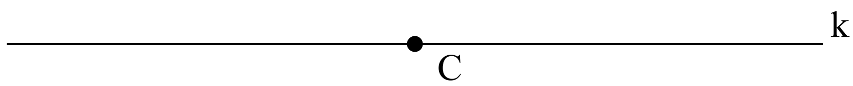
Construction 5 **Parallel Through a Point Not on a Line** - Given a point outside a line, construct the parallel to the given line through the given point.

Construction 6 **Quadrilateral With Parallel Sides** – Construct a quadrilateral with one pair of parallel sides of lengths a and b.

a

b

Construction 7 **Perpendicular Through a Point on a Line** - Given a point on a line, construct the perpendicular to the line at the given point.



Construction 8 **Perpendicular Through a Point Not on a Line** - Given a point outside a line, construct the perpendicular to the line from the given point.