Logarithms

$y = \log_b x$ is equivalent to $x = b^y$

ex: $\log_5 125 = 3$ because $5^3 = 125$

$\log x = \log_{10} x$ common log

Natural Logs

$y = \ln x$ is equivalent to $x = e^y$

ex: $\log e^4 = 4$ because $e^4 = e^4$

$\ln x = \log_e x$ natural log where $e = 2.71828\ldots$

Properties	Examples	Properties	Examples
$\log_b b = 1$	$\log_3 3 = 1$	$\ln e = 1$	
$\log_b b^x = x$	$\log_5 5^7 = 7$	$\ln e^x = x$	$\ln e^3 = 3$
$\log_b 1 = 0$		$\ln 1 = 0$	
$b^{\log_b x} = x$	$4^{\log_4 9} = 9$	$e^{\ln x} = x$	$e^{\ln 6} = 6$
$\log_b x^r = r \log_b x$	$\log 5^2 = 2 \log 5$	$\ln x^r = r \ln x$	$\ln x^3 = 3 \ln x$
$\log_b (xy) = \log_b x + \log_b y$	$\log(2x) = \log 2 + \log x$	$\ln xy = \ln x + \ln y$	$\ln 6 = \ln 2 + \ln 3$
$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$	$\log \frac{3}{7} = \log 3 - \log 7$	$\ln \frac{x}{y} = \ln x - \ln y$	$\ln \frac{2}{5} = \ln 2 - \ln 5$

Change of base formula:

$$\log_b a = \frac{\log a}{\log b}$$

The domain of $\log_b x$ is $x > 0$

Solving exponential equations: If $b^x = b^y \Leftrightarrow x = y$

If $M = N \Leftrightarrow \log M = \log N$

Models using Logarithms/Exponents

Simple Interest $I = Prt$

Compound Interest $A = P\left(1+\frac{r}{n}\right)^{nt}$

Continuous Compounding $A = Pe^{rt}$

I = interest earned
P = principal (initial amount of money)
r = interest rate (as a decimal)
t = time in years
A = amount of money after t yrs.
n = number of times it is compounded per year

Effective Rate of Interest (r_e)

compounded n times/year $r_e = \left(1+\frac{r}{n}\right)^n - 1$

compounded continuously $r_e = e^r - 1$

Present Value (PV)

$$PV = A\left(1+\frac{r}{n}\right)^{-nt}$$

if continuous $PV = Ae^{-rt}$

Growth where $k > 0$ $A(t) = A_0 e^{kt}$

Decay where $k < 0$

A = Amount
A_0 = initial amount at time $t = 0$

Newton's Law of Cooling

$u(t) = T + (u_0 - T)e^{kt}$ where $k < 0$

u = Tempurature at any given time
T = constant temperature of the surrounding medium
u_0 = initial temp. of object

Logistic Model

Describes situations where growth or decay is limited.

$$P(t) = \frac{c}{1 + ae^{-bt}}$$

$P(t)$ = Population after time t

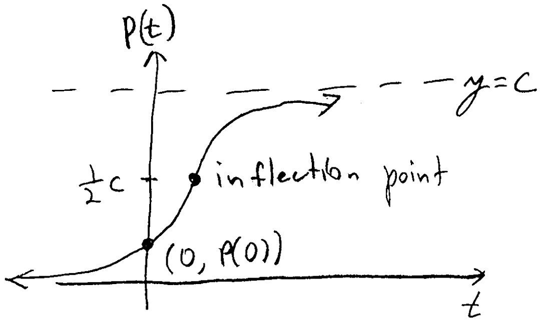
a, b, c are constants

$c > 0$, c is the carrying capacity for growth models

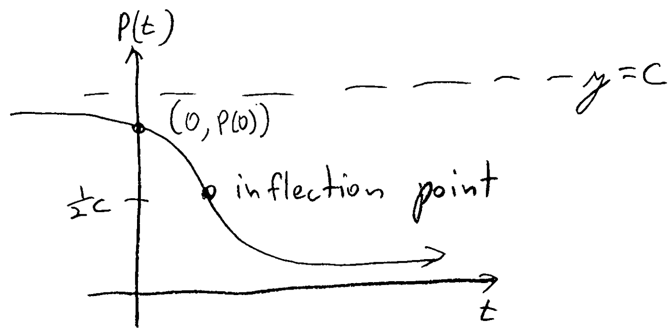
If $b > 0$, then growth model

If $b < 0$, then decay model

$|b|$ is the growth rate for $b > 0$ and decay rate for $b < 0$



Growth



Decay